Observing quantum trajectories of a superconducting qubit

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Quantum Electronics group
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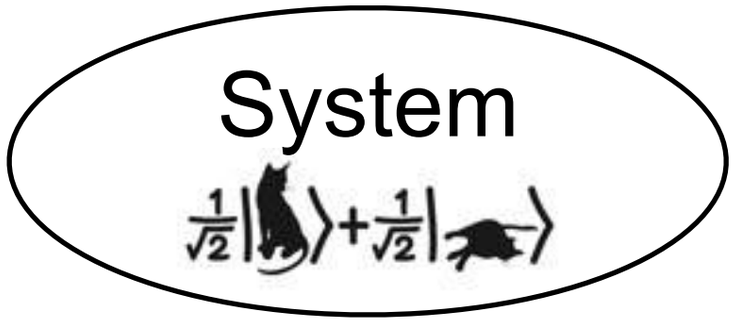
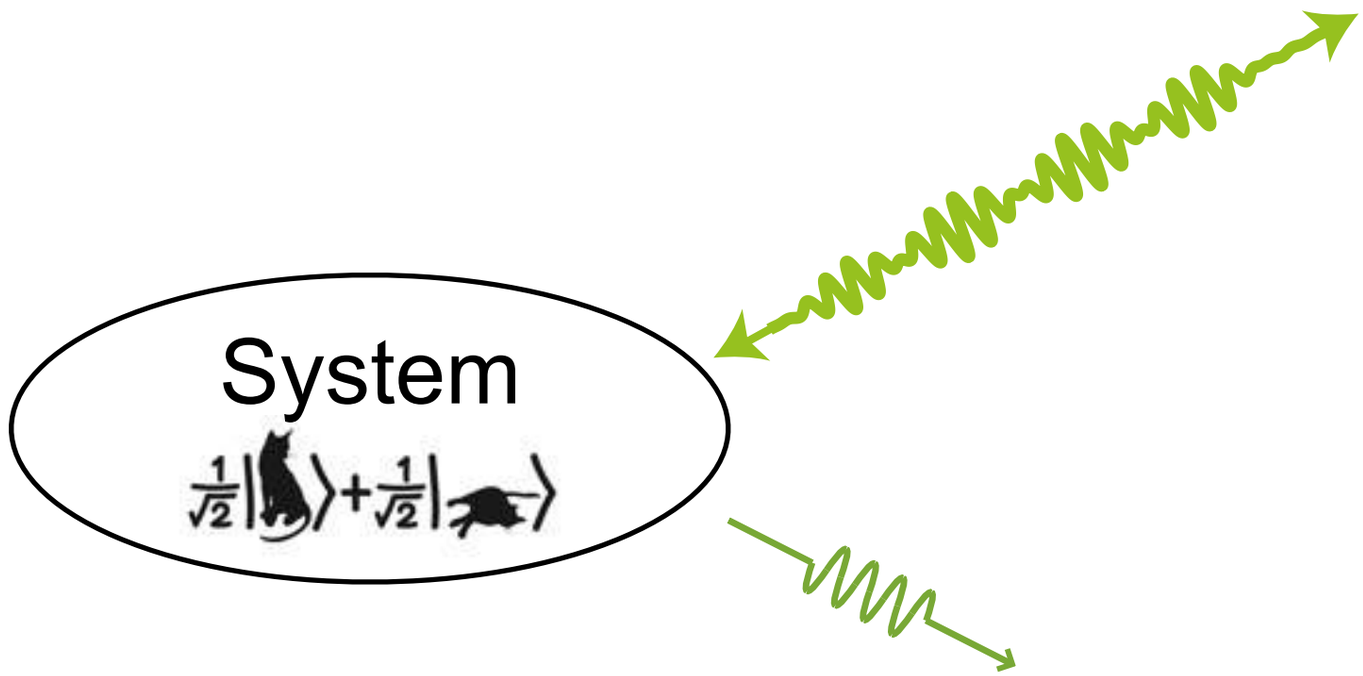
Quantum trajectories, parameter and state estimation
Toulouse 2017

Quantum trajectory of an open quantum system



Quantum trajectory of an open quantum system



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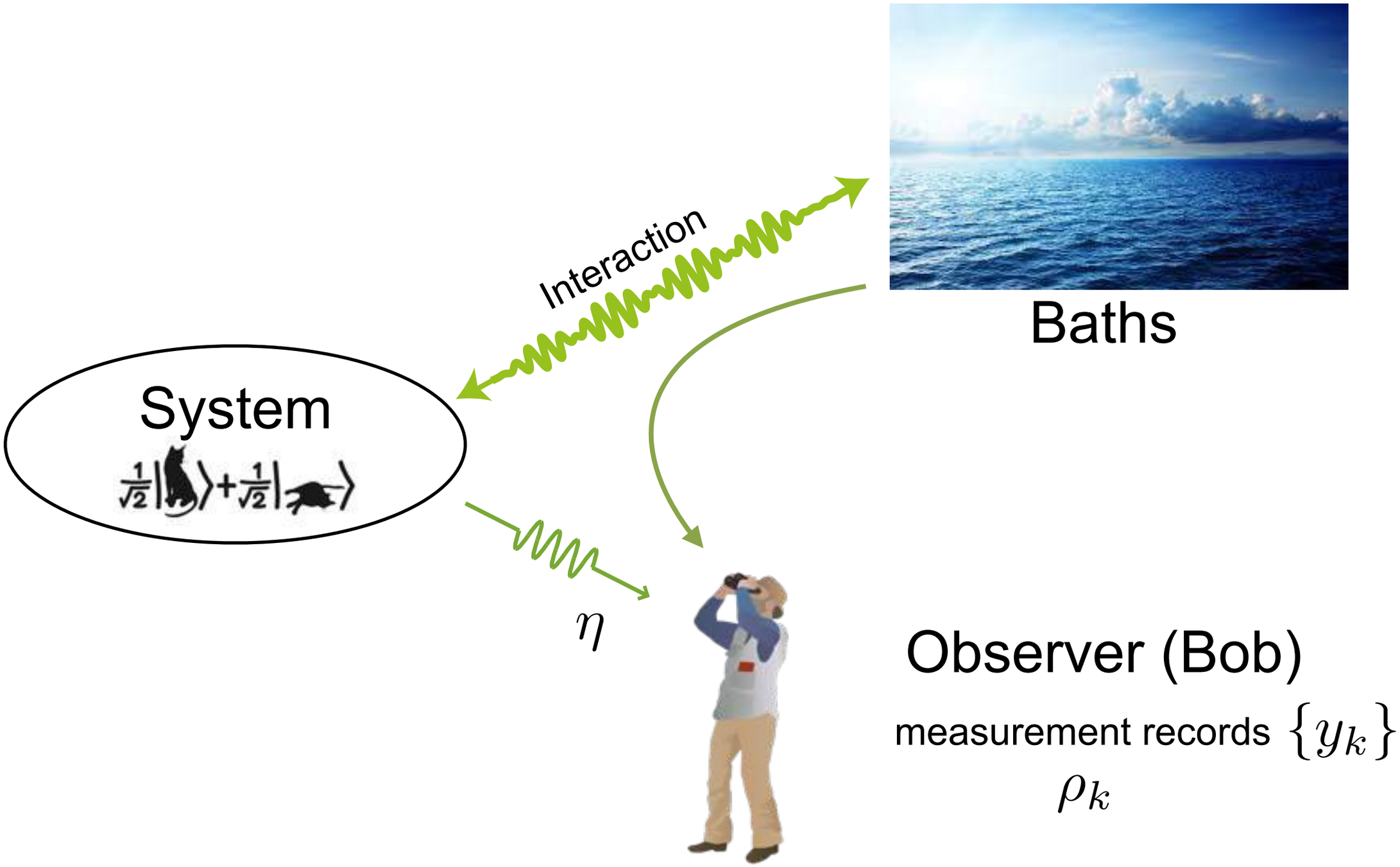

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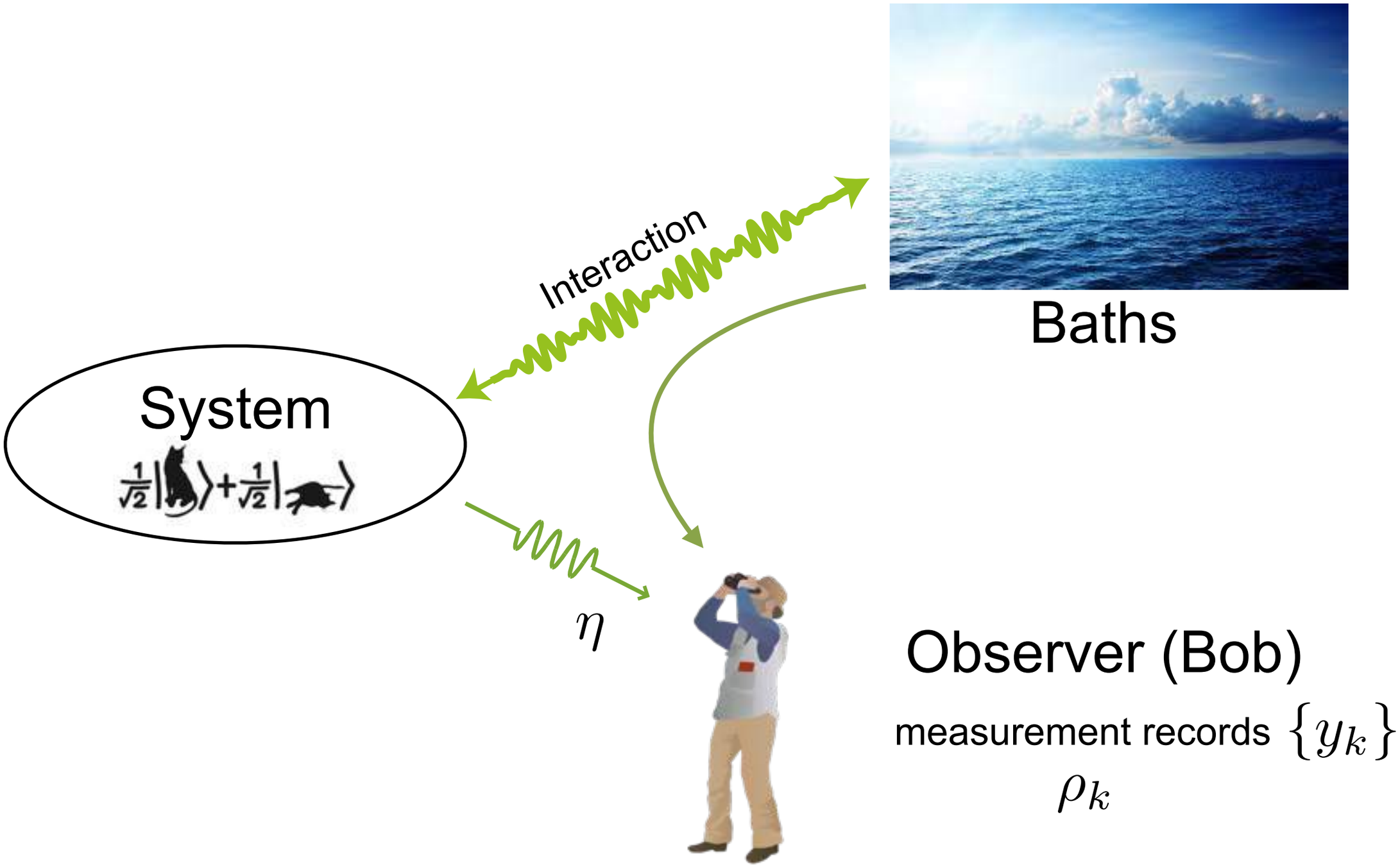
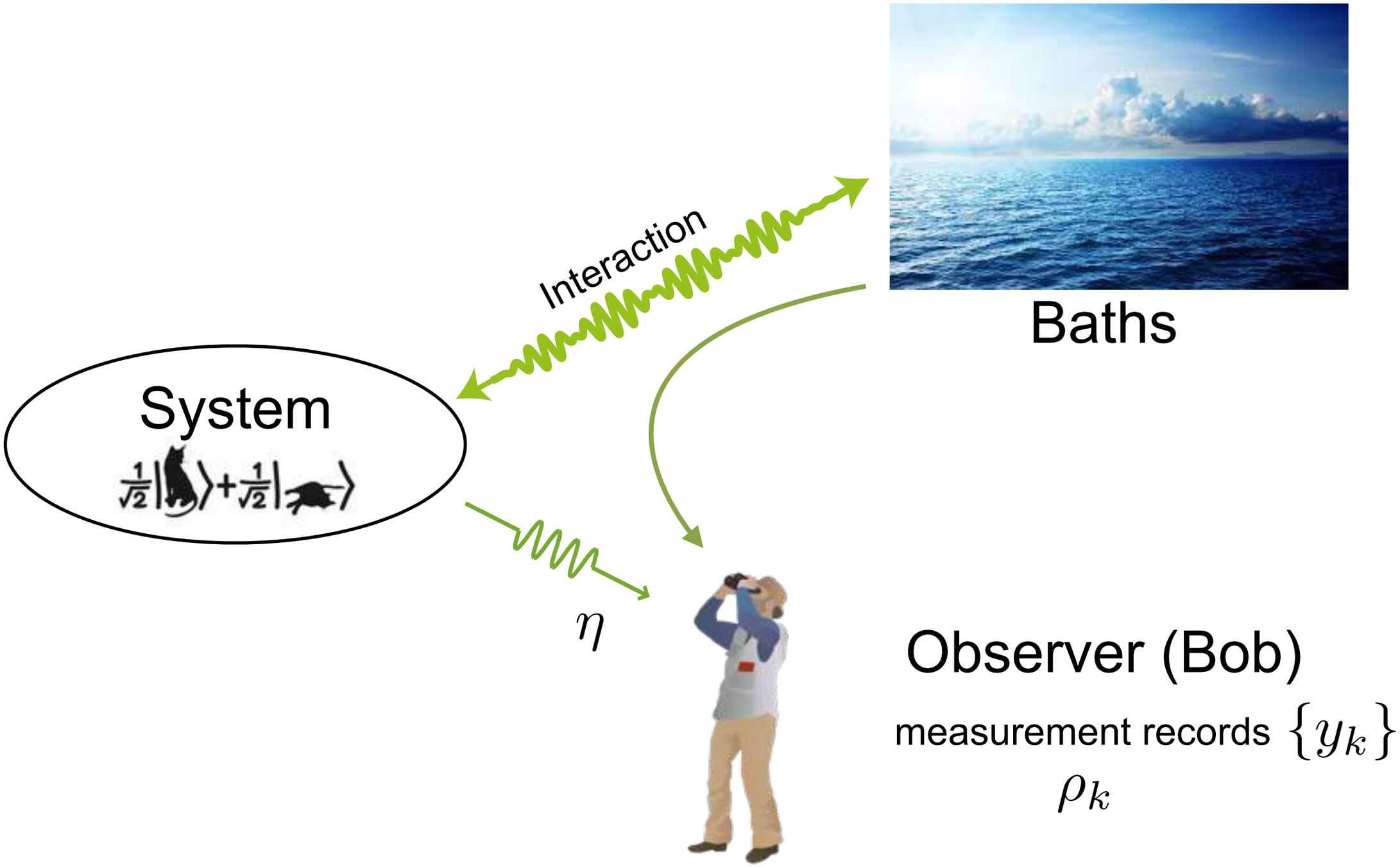


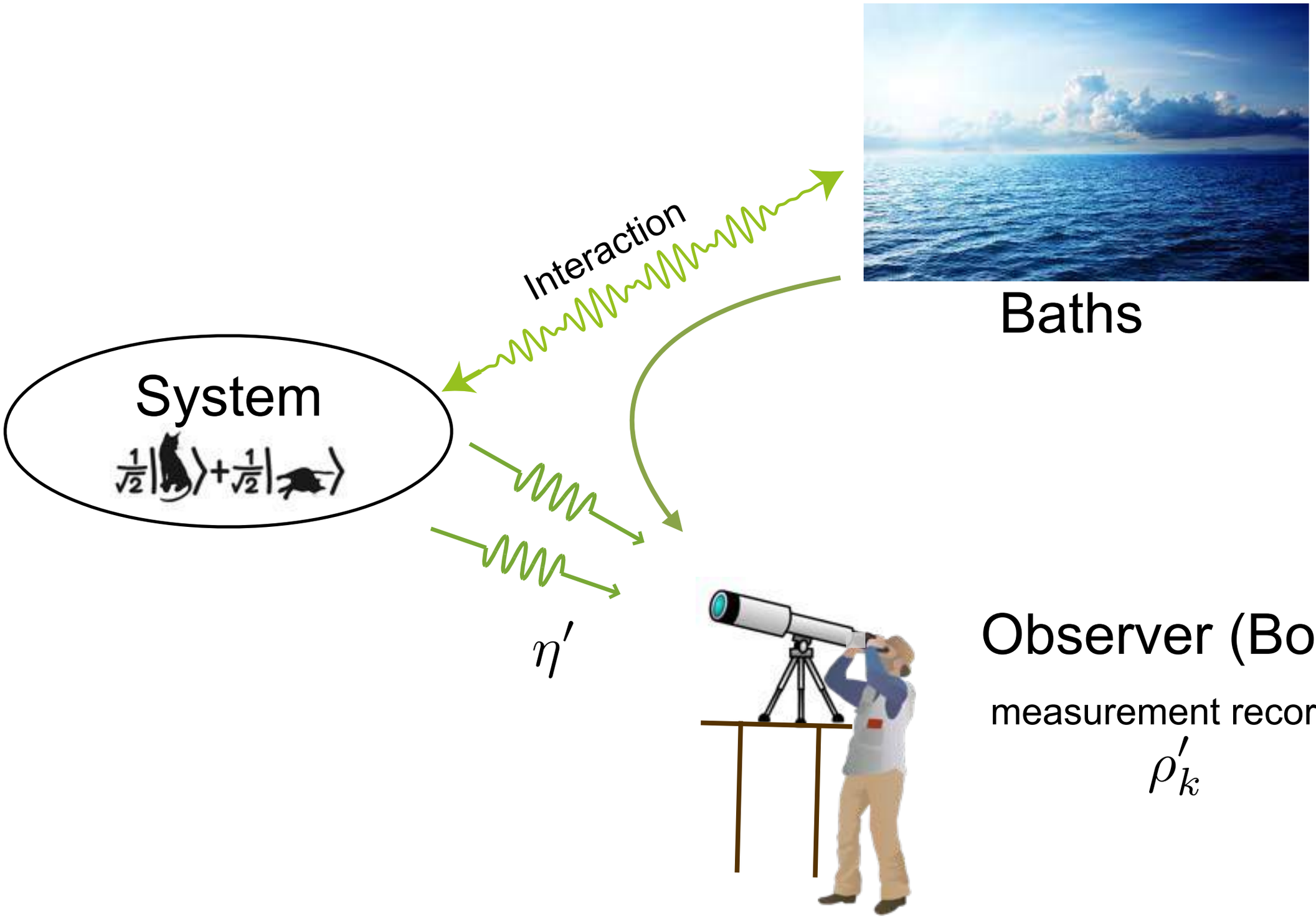
$$\mathbb{P}(\langle A\rangle = a|\{y_1, ..., y_{k-1}\}) = \mathrm{Tr}(\Pi_a \rho_k)$$

Quantum trajectory of an open quantum system



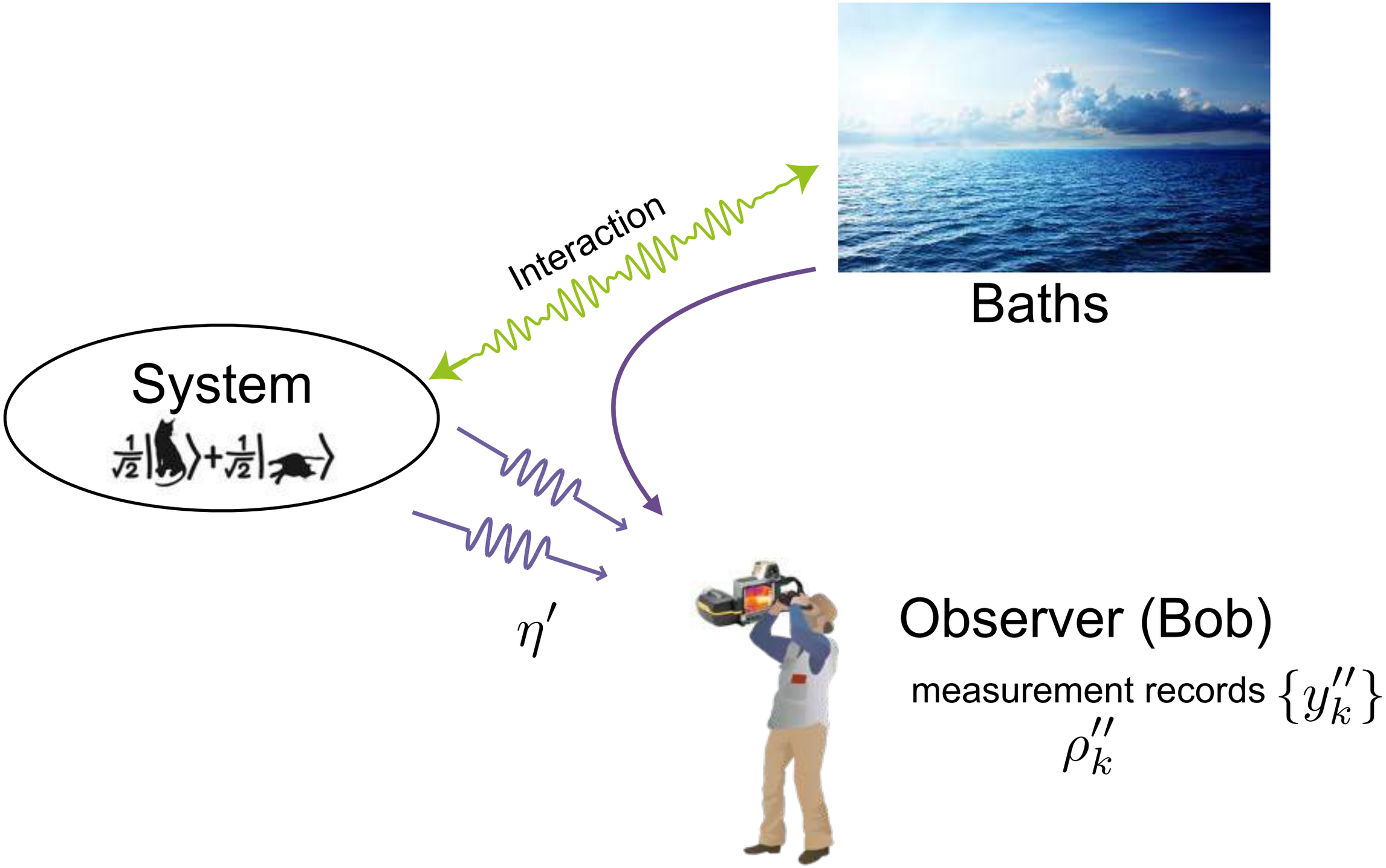
Quantum trajectory = $\{\rho_k\}_k$

Quantum trajectory of an open quantum system



The quantum trajectories depend on the observation

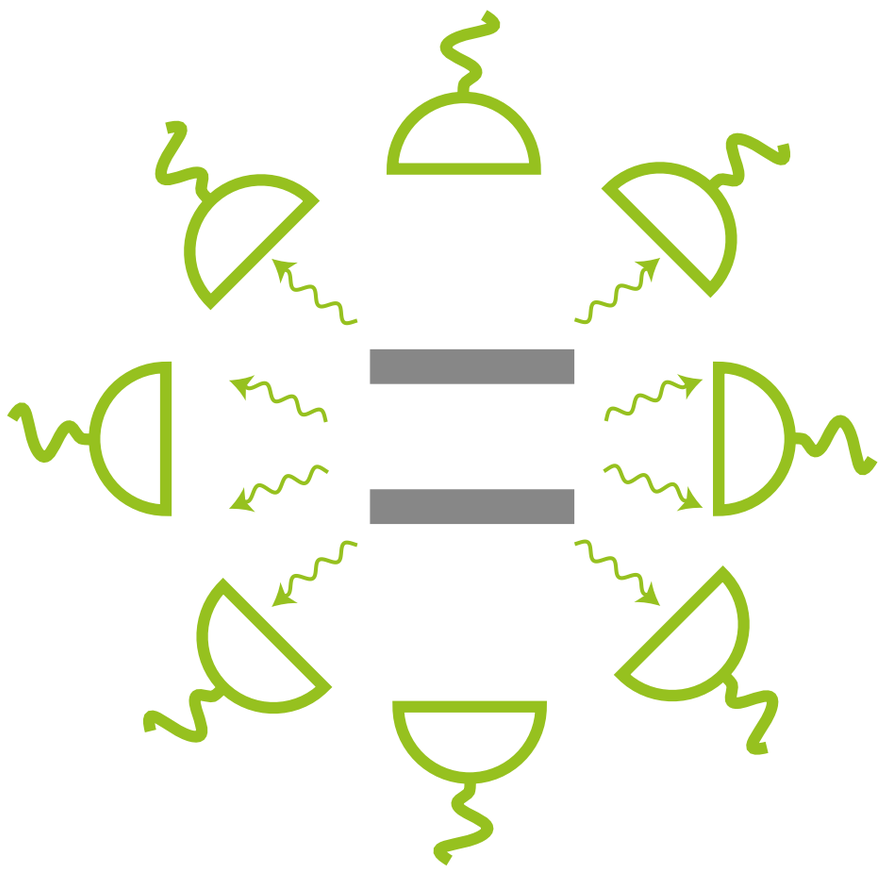
Quantum trajectory of an open quantum system



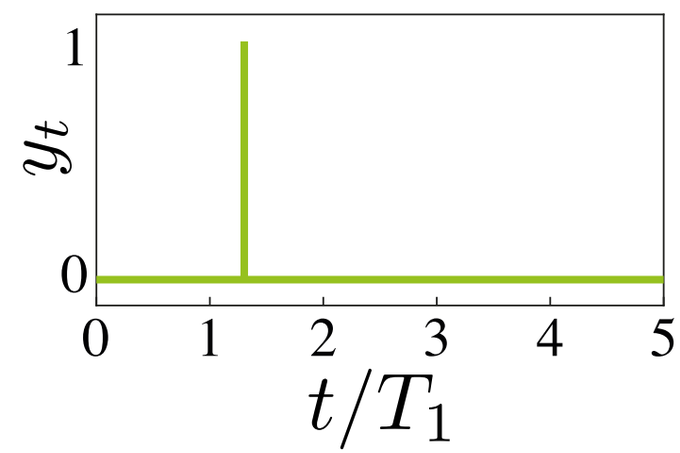
The quantum trajectories depend on the observation

Example 1 : Quantum jumps of a qubit

System = qubit



Detector = perfect photon counter



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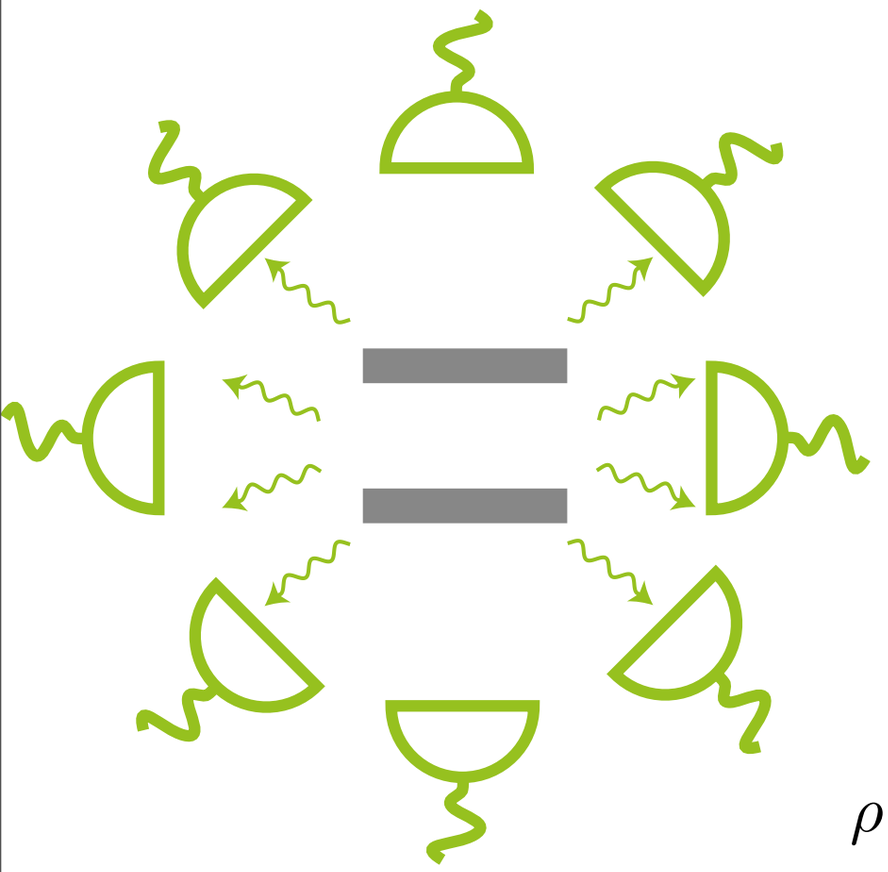
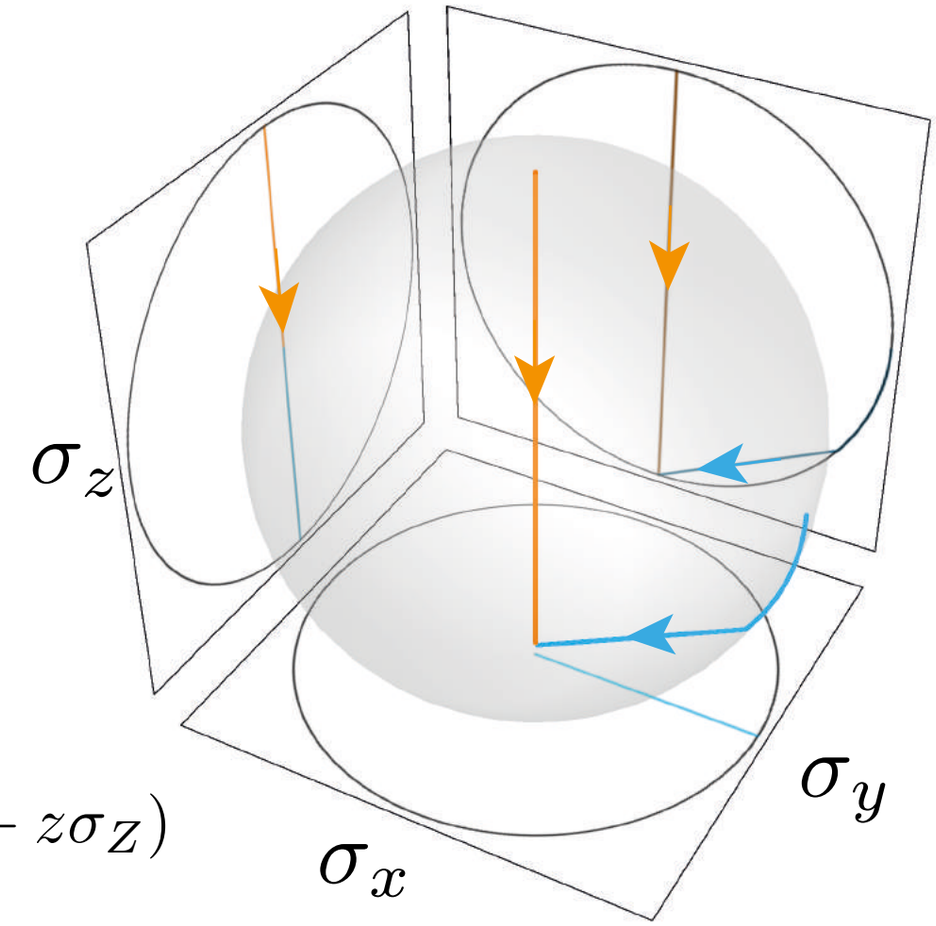
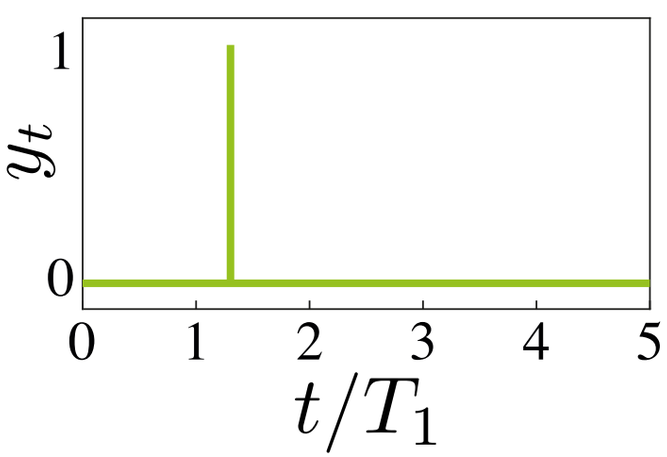


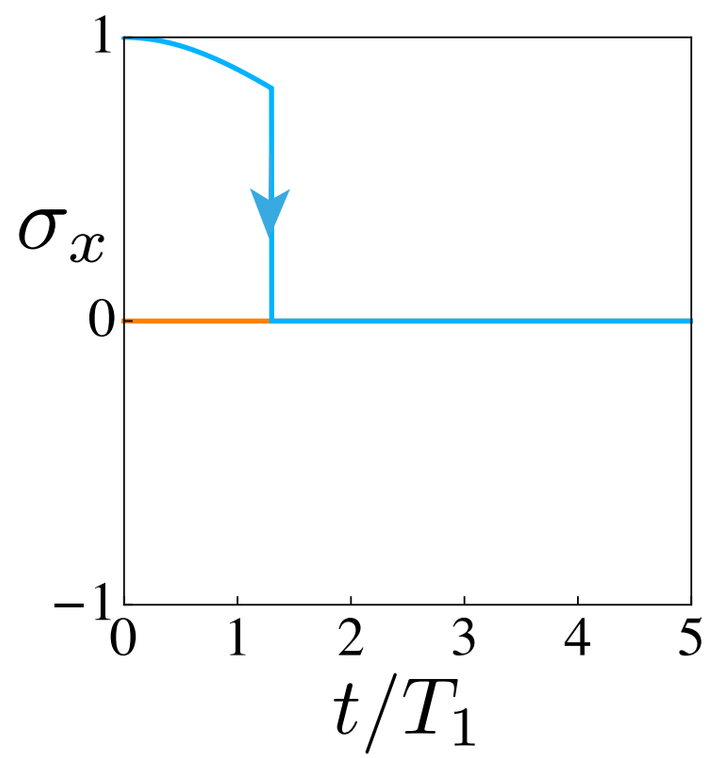
Starting from $|e\rangle$

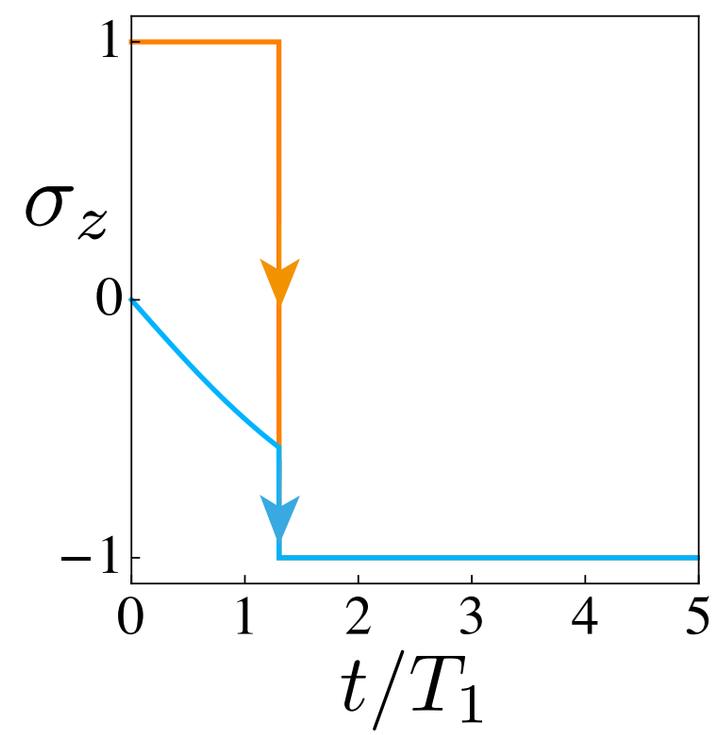
Starting from $\frac{|g\rangle + |e\rangle}{\sqrt{2}}$



$$\rho = \frac{1}{2}(\mathbf{1} + x\sigma_X + y\sigma_Y + z\sigma_Z)$$



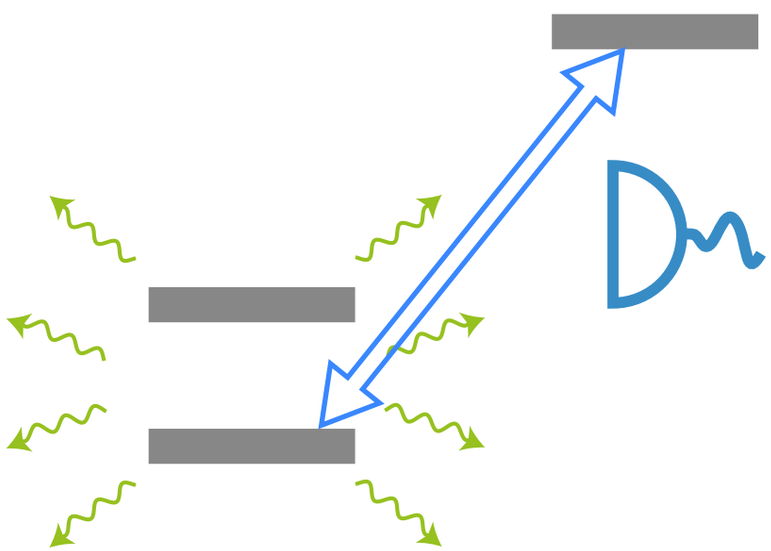




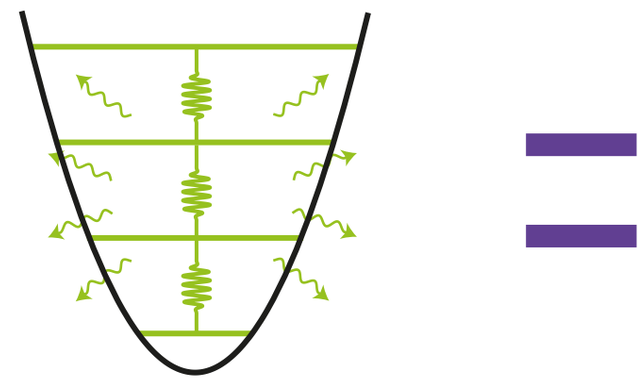
Note: purity of state is 1 only for perfect detectors

Quantum jumps

hard to collect → use an ancillary detector

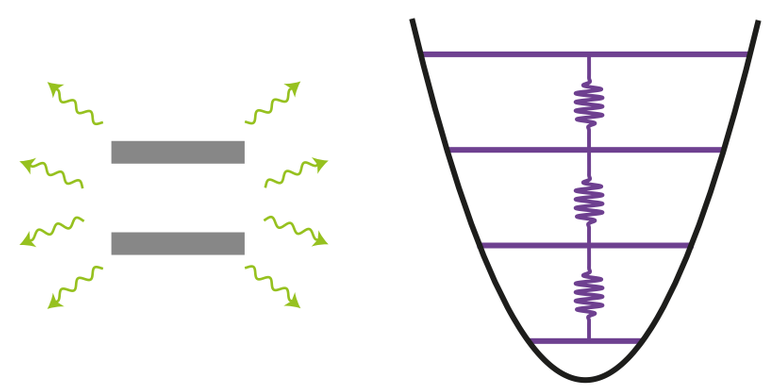


since 1986 in trapped ions
[Wineland group, Boulder
Dehmelt groupe, Seattle
Toschek group, Hambourg]



$$H_{\mathrm{coupl}} = \hbar\chi a^{\dagger} a \frac{\sigma_z}{2}$$

Rydberg atom probing cavity jumps
[Haroche group, Paris (2007)]



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Cavity probing qubit jumps
[Siddiqi group, Berkeley (2011)]

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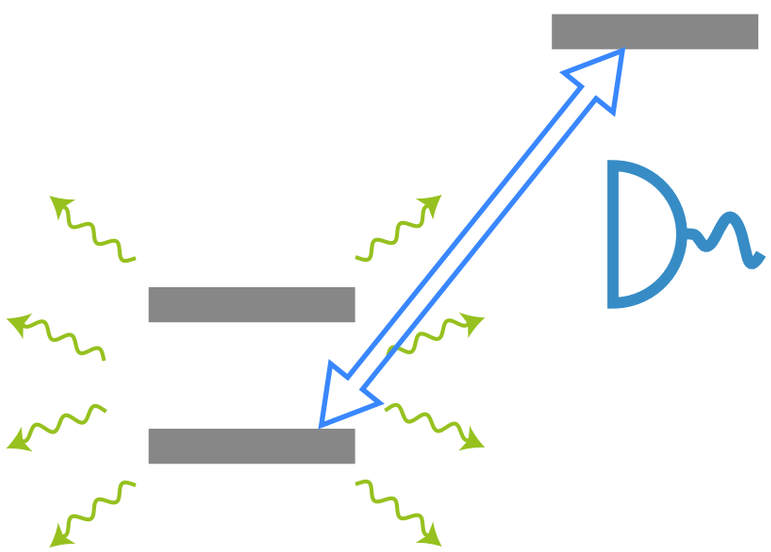

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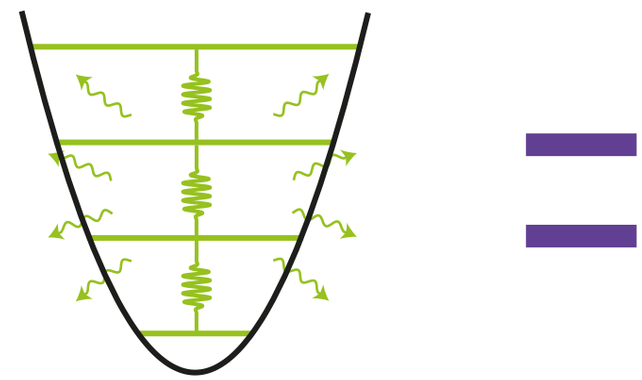

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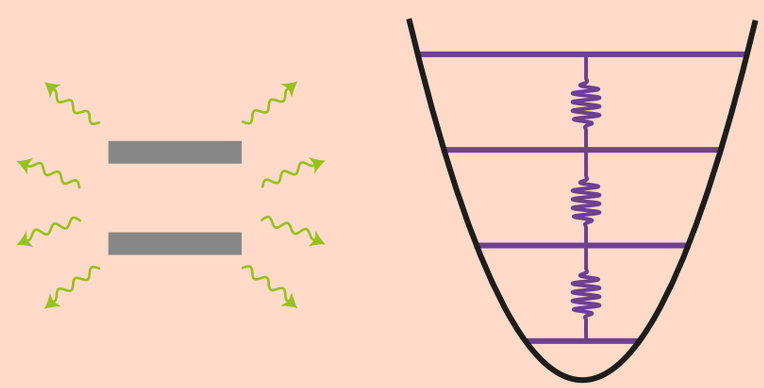

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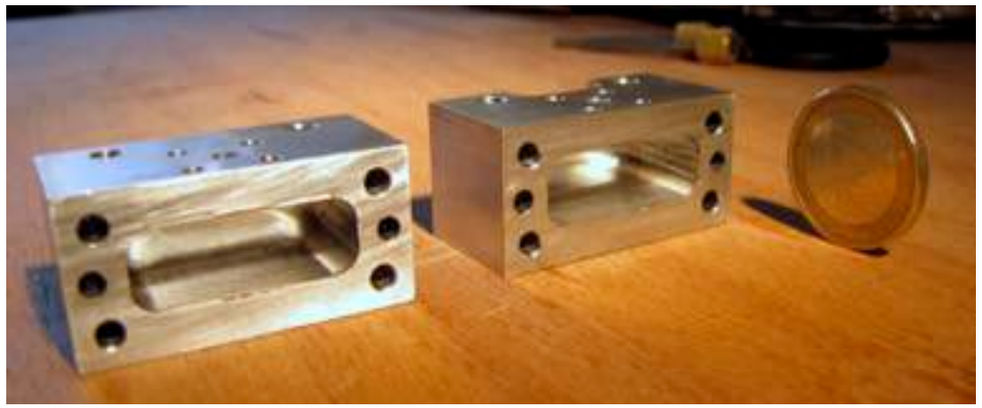
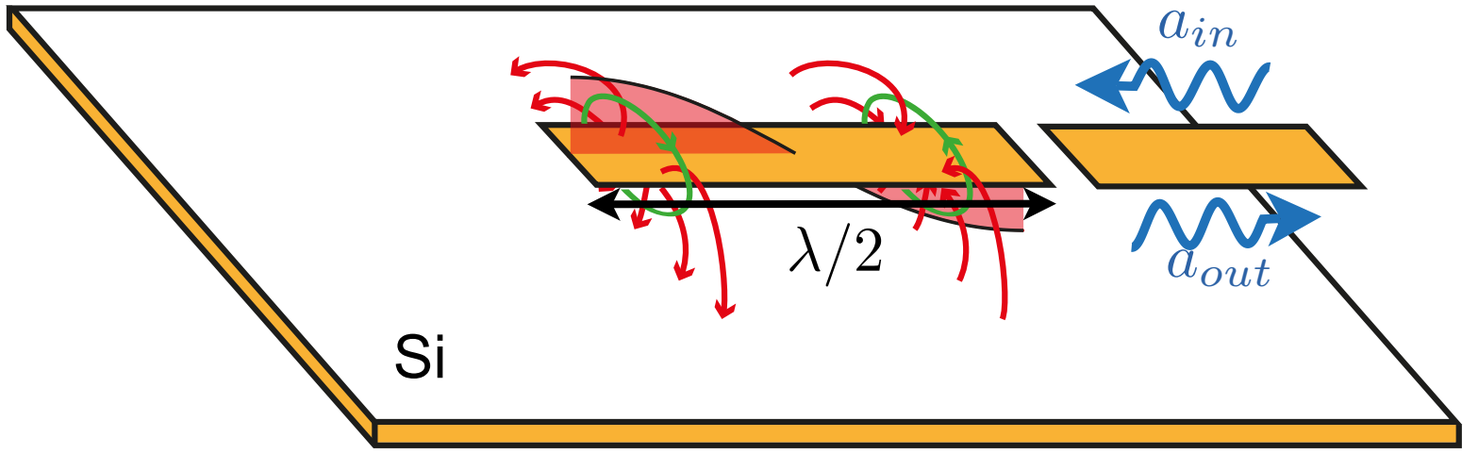
Superconducting Circuits

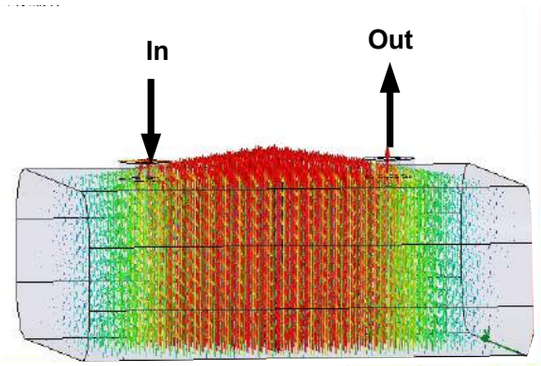


1st mode : 7.63 GHz

$$Q \approx 10^6$$

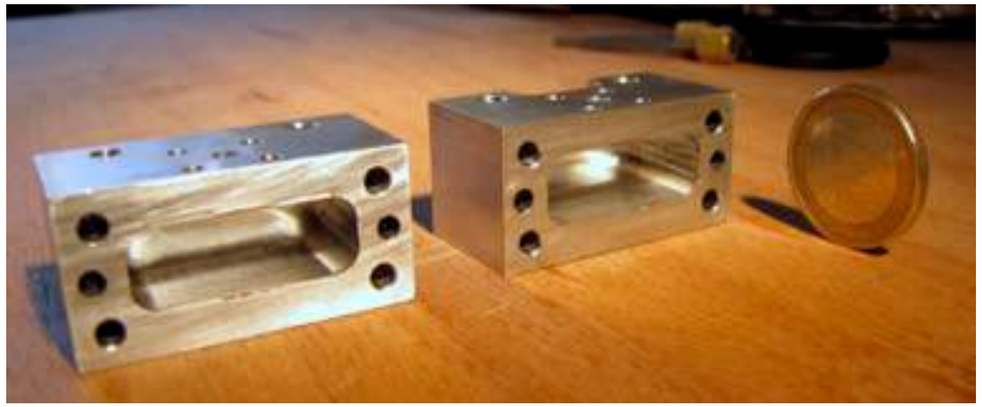


$$\omega_0 = 1/\sqrt{LC}$$



$$k_B T \ll \omega_0$$
$$T \sim 20 \text{ mK}$$

Superconducting circuit



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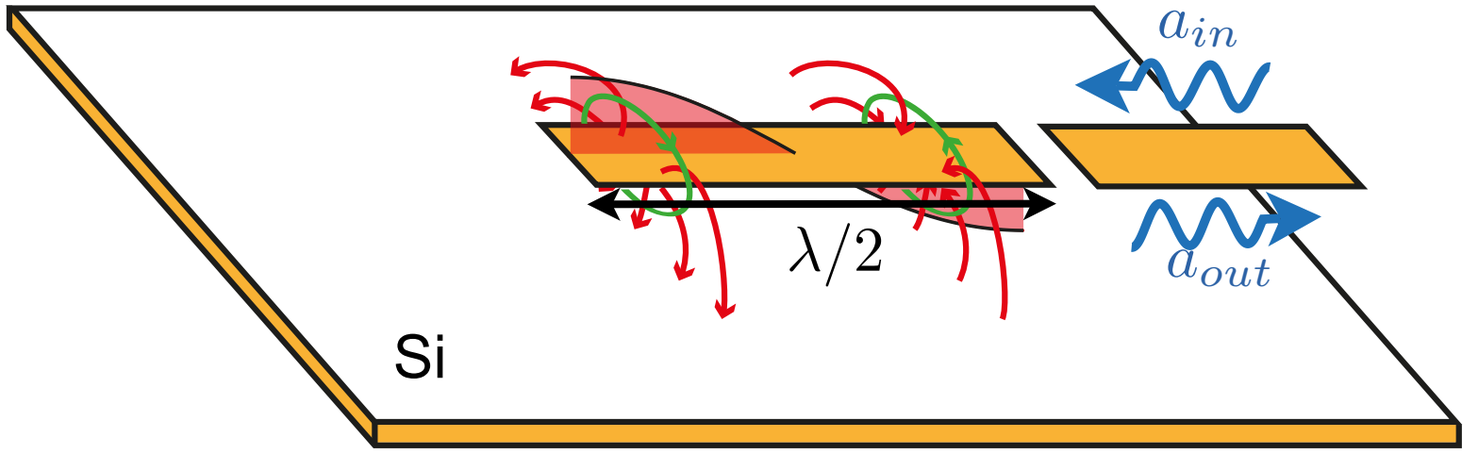

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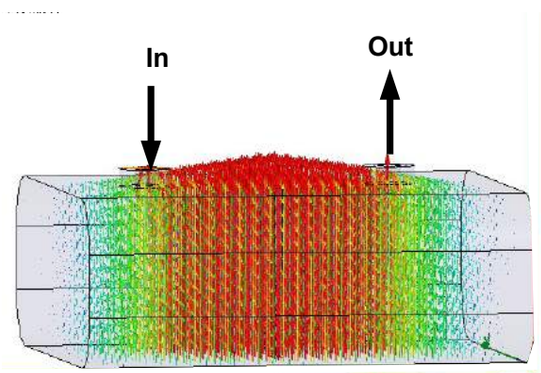

$$k_B T \ll \omega_0$$
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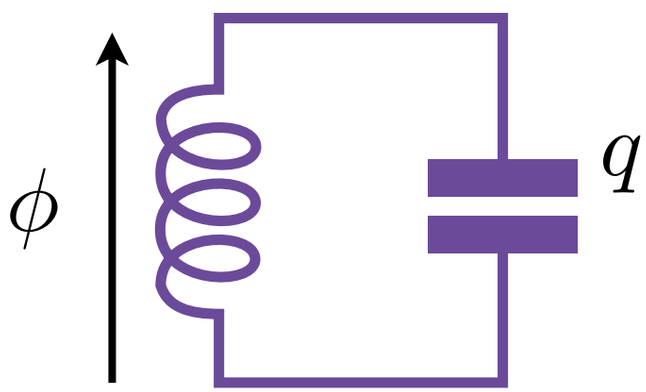
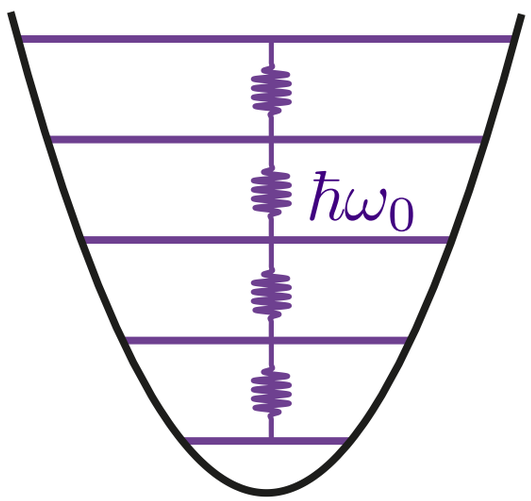
dissipationless LC circuit ...



$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

$$[\hat{\phi}, \hat{q}] = i\hbar$$

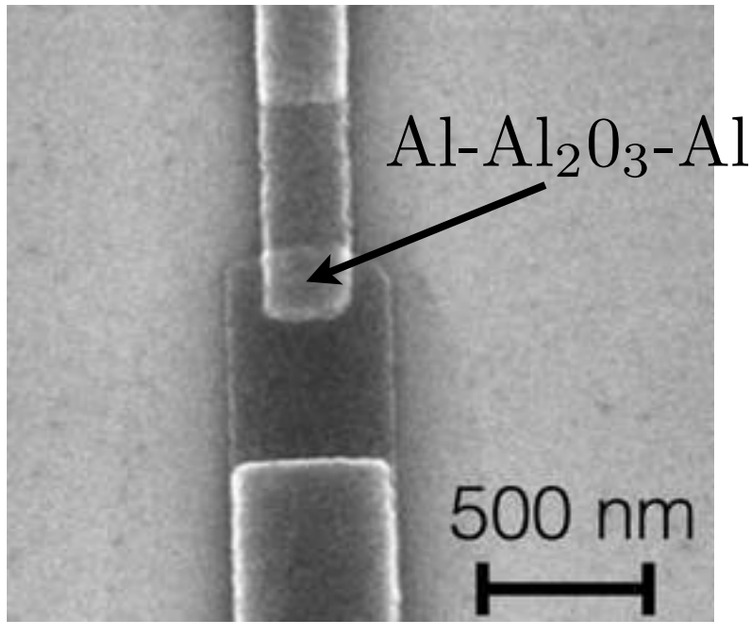
... canonically quantized



$$\hat{H} = \hbar\omega_0(\hat{a}^\dagger \hat{a} + \frac{1}{2})$$

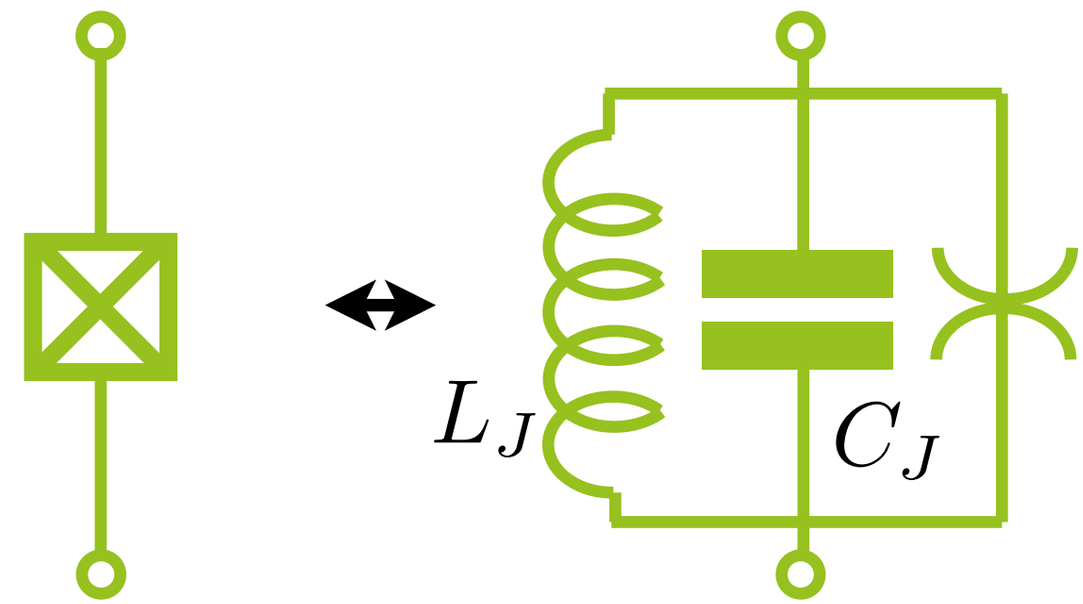
Superconducting Circuits

Josephson Junction



Non linear dissipationless inductor

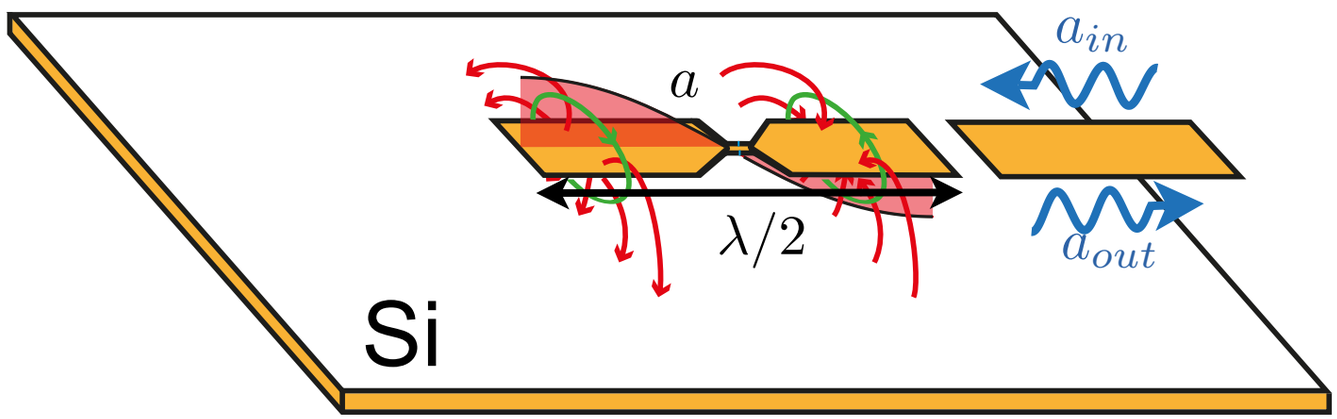
dissipation-less non linear LC circuit

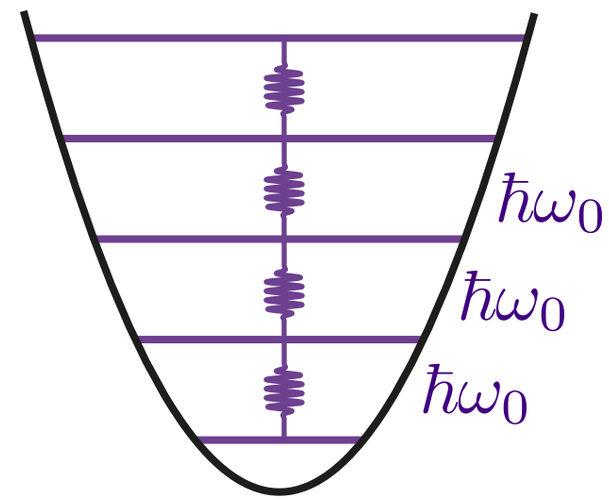


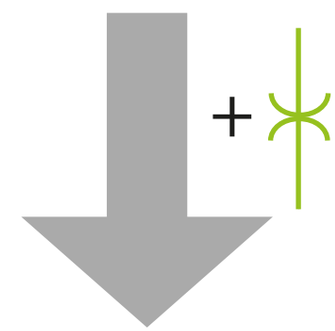
$$\hat{H} = \frac{\hat{q}^2}{2C_J} - E_J \cos \frac{\hat{\phi}}{\hbar/2e} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\phi}^2}{2L_J} + H_{\text{non-lin}}(\hat{\phi})$$

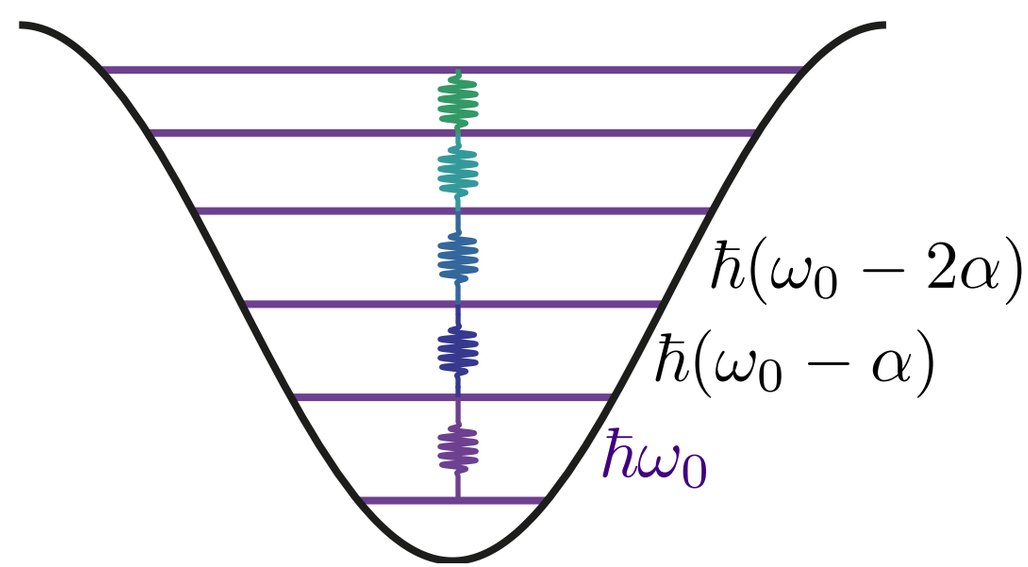
Superconducting circuits with Josephson junctions

dissipation-less non linear LC circuit





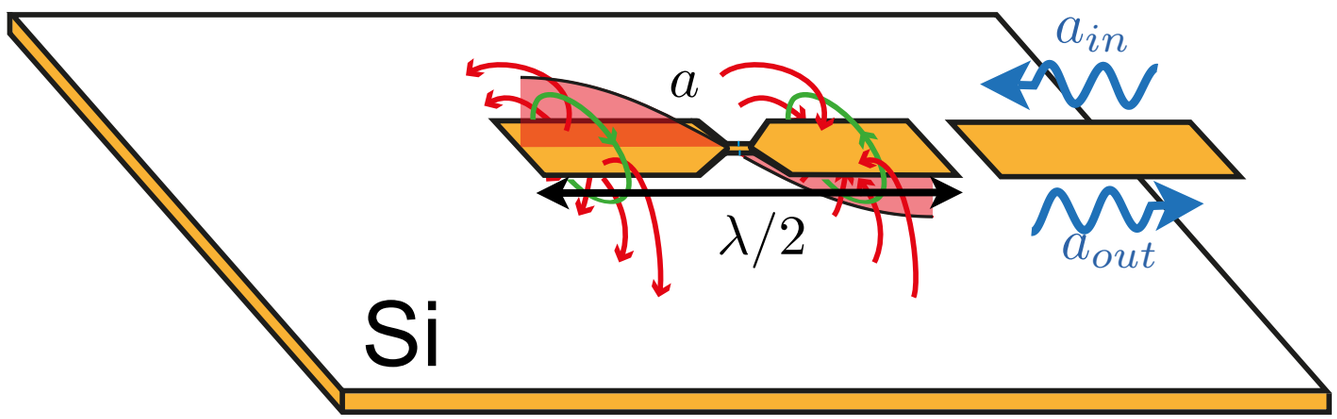


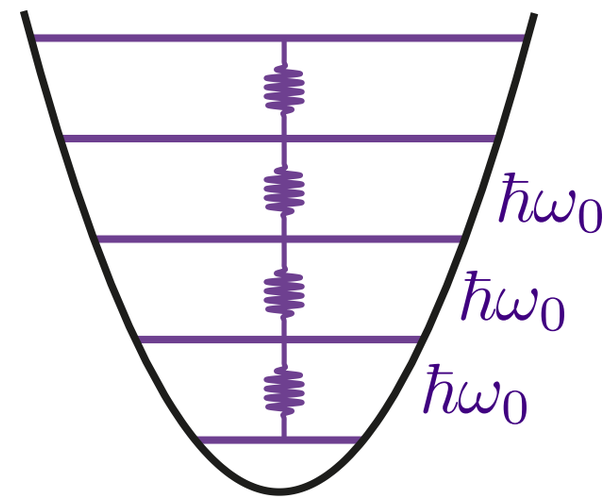


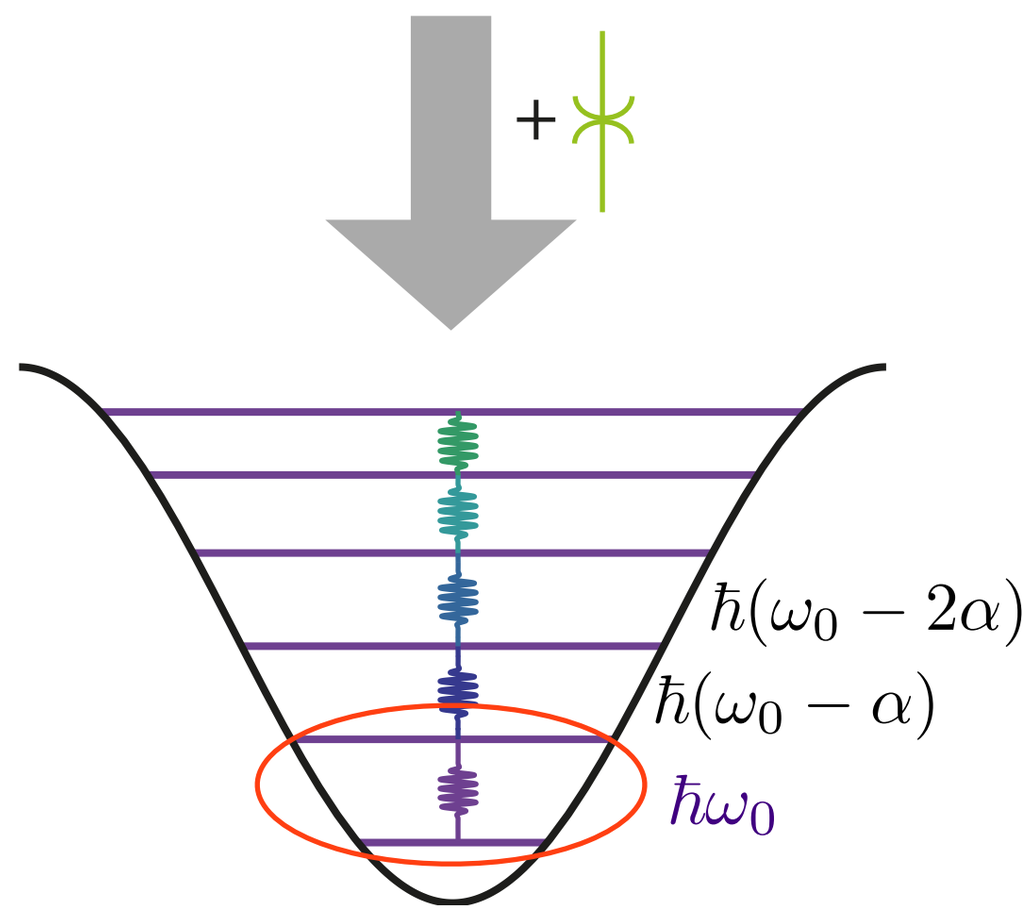
transitions observed in 1980's [Berkeley & Saclay]
strong coupling regime of CQED in 2004 [Yale]

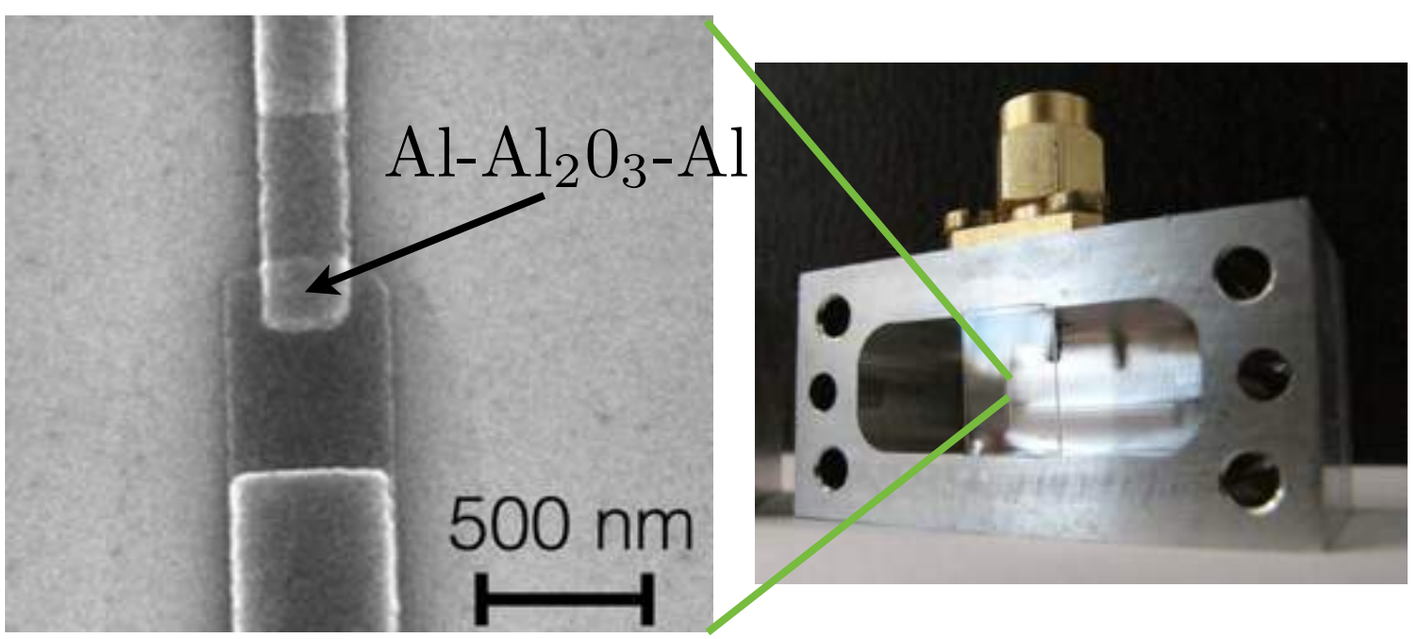
Superconducting circuits with Josephson junctions

dissipation-less non linear LC circuit







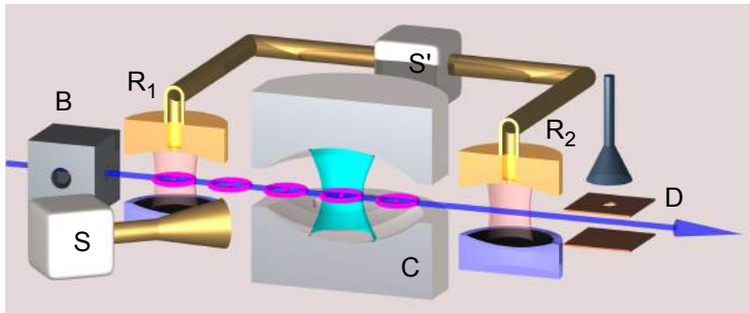


3D transmon

transitions observed in 1980's [Berkeley & Saclay]
strong coupling regime of CQED in 2004 [Yale]

Superconducting circuits with non linear systems

Rydberg atoms



[pic from CQED group, College de France Paris]

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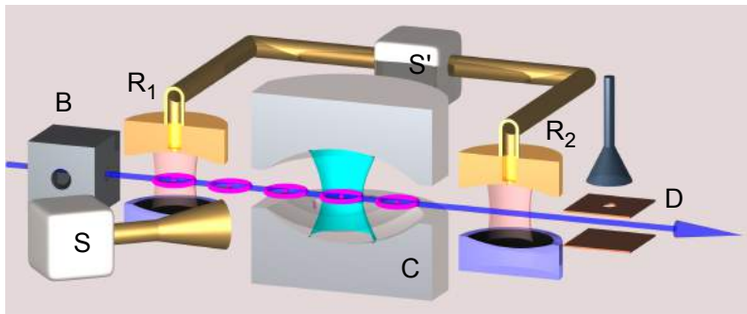

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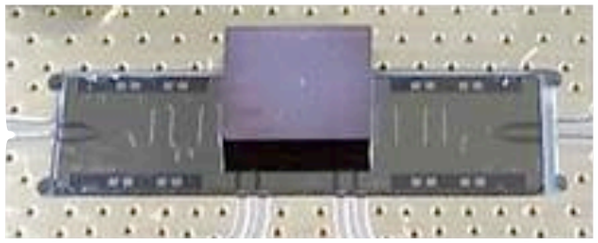
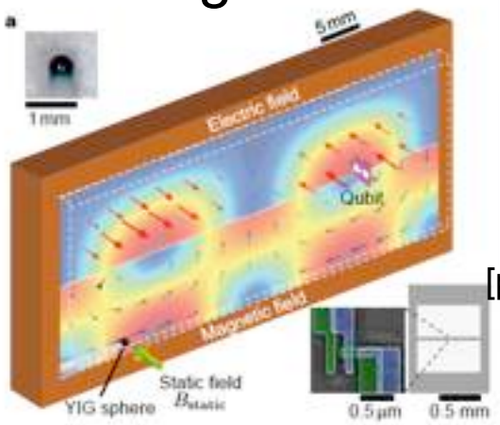
NV centers



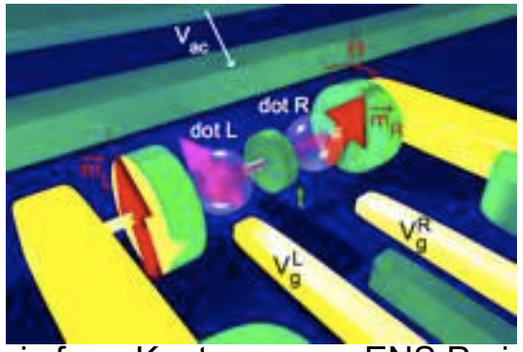
[pic from Quantronics group, CEA Saclay]

Ferromagnetic magnons



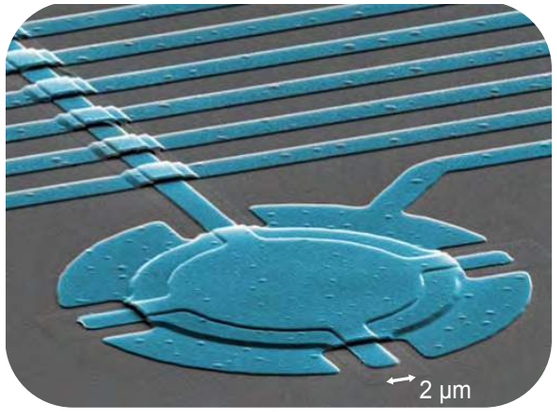
[pic from Nakamura-Usami group, Univ. Tokyo]

Spins in CNT



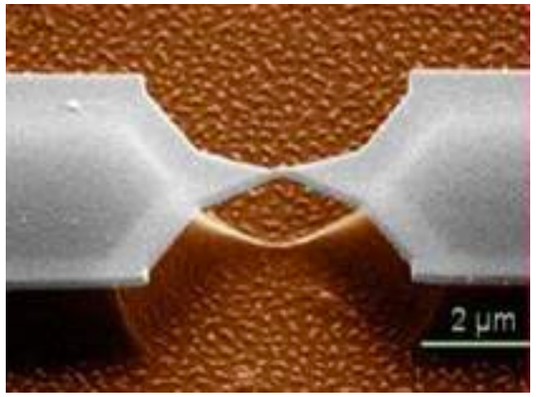
[pic from Kontos group, ENS Paris]

Metallic membrane



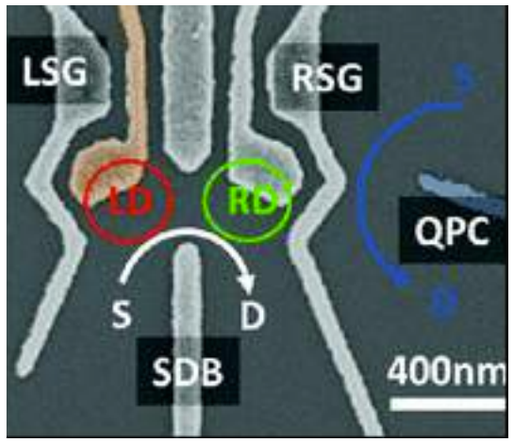
[pic from Lehnert group, JILA Boulder]

Andreev Bound States



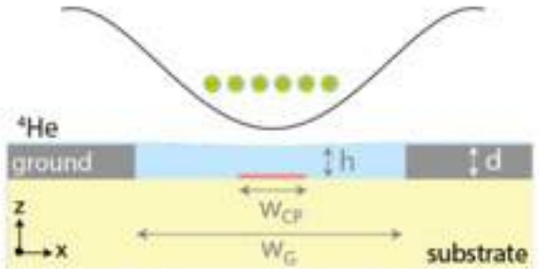
[pic from Quantronics group, CEA Saclay]

Semiconductor quantum dots



[pic from Wallraff group, ETH Zurich]

Electrons on ^{4}He

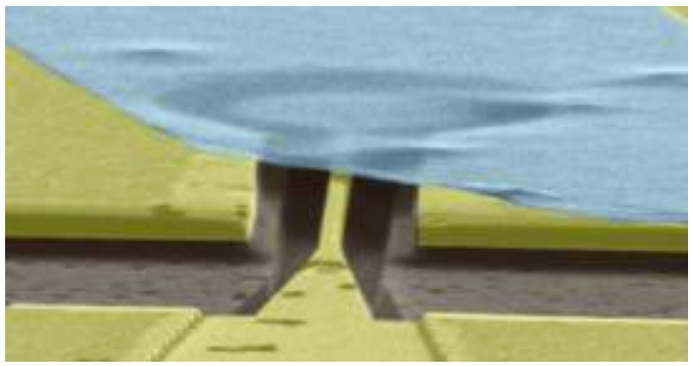


[pic from Shuster group, Chicago]

Propagating phonons

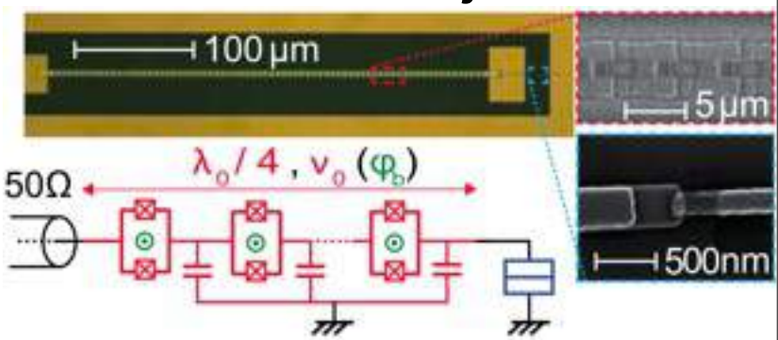
[pic from Delsing group, Chalmers UT]

Graphene membrane



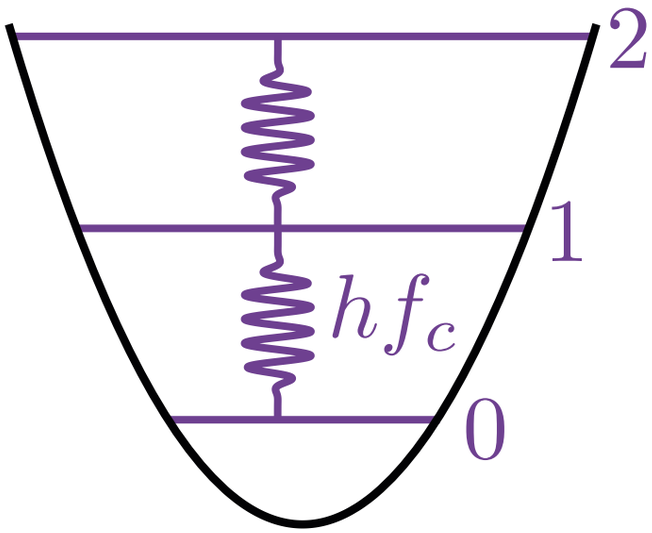
[pic from Steele group, TU Delft]

DC biased junction



[pic from Saclay group]

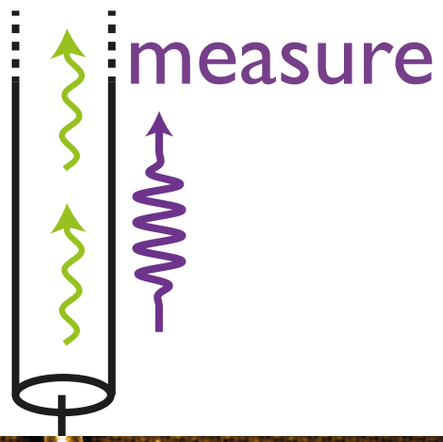
3D transmon architecture

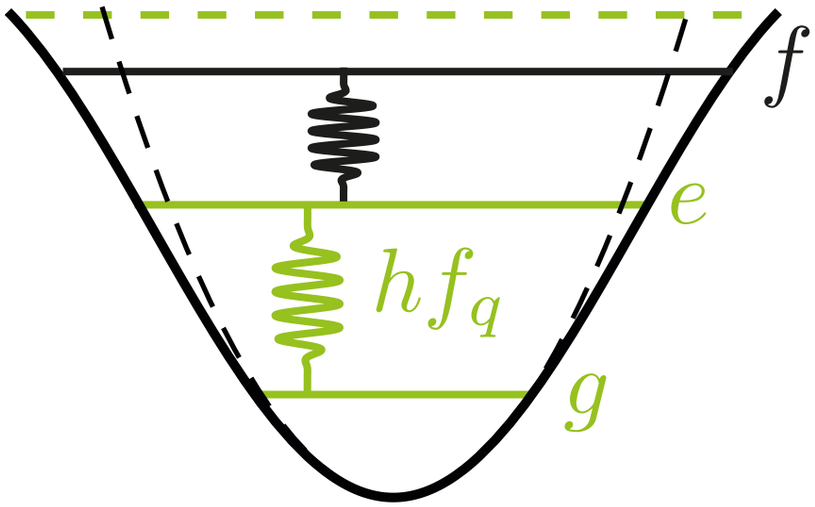


$$H_c = hf_c(a^\dagger a + \frac{1}{2})$$

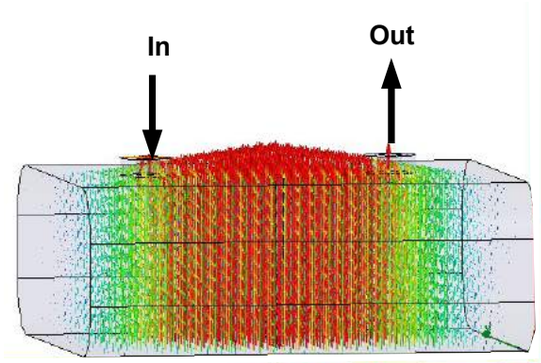
$$H_{\text{disp}} = h\chi \frac{\sigma_z}{2} a^\dagger a$$

$$\frac{\chi}{2\pi} = 4 \text{ MHz}$$





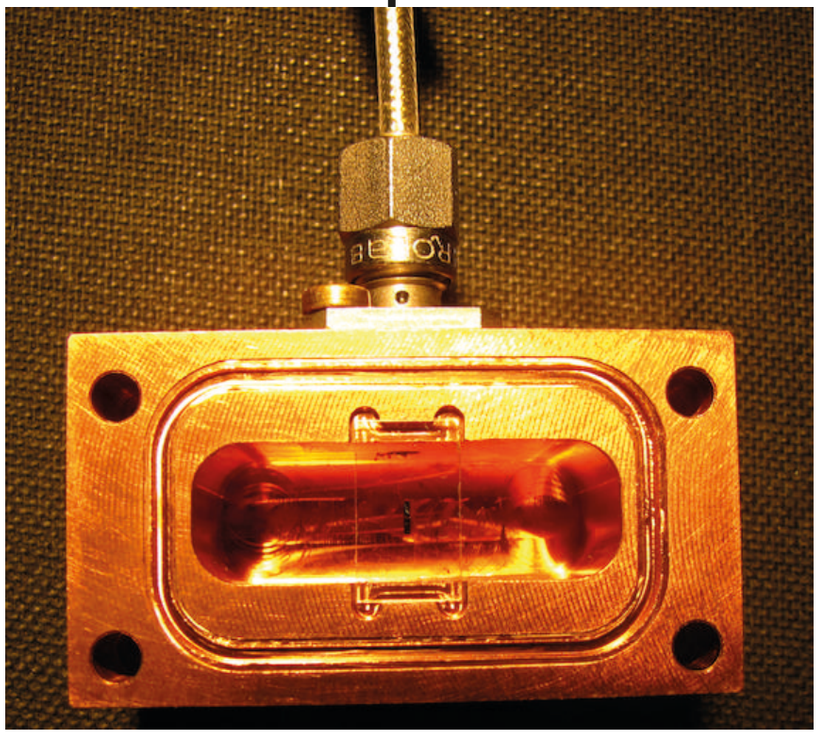
$$H_q = hf_q \frac{\sigma_z}{2}$$

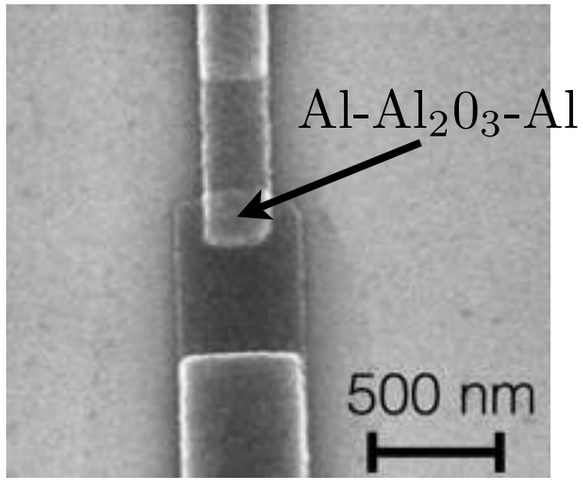


$$f_c = 7.8 \text{ GHz}$$

$$k_B T \ll hf$$

$$T = 20 \text{ mK}$$

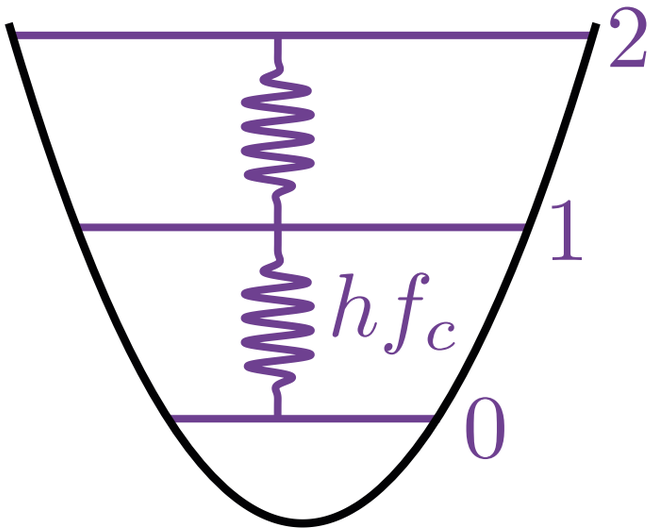




$$f_q = 5.4 \text{ GHz}$$

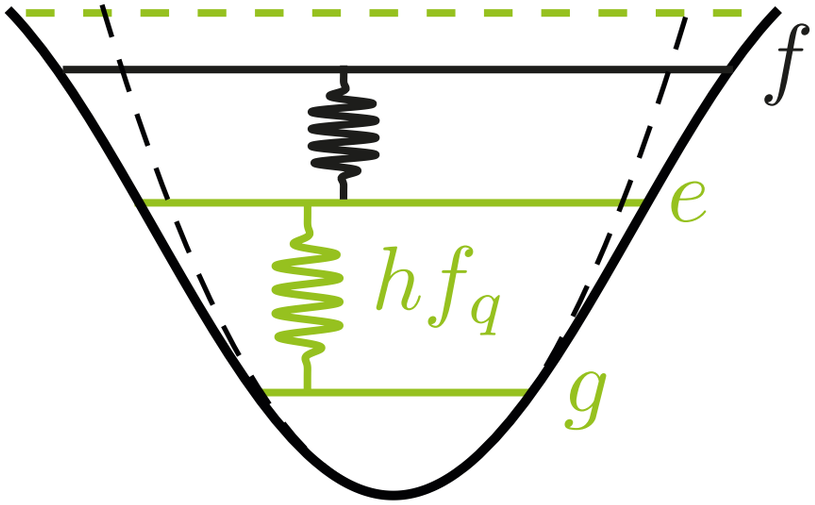
$$T_1 = 10\ \mu\text{s}$$

3D transmon architecture



$$H_{\text{disp}} = h\chi \frac{\sigma_z}{2} a^\dagger a$$

$$\frac{\chi}{2\pi} = 4 \text{ MHz}$$

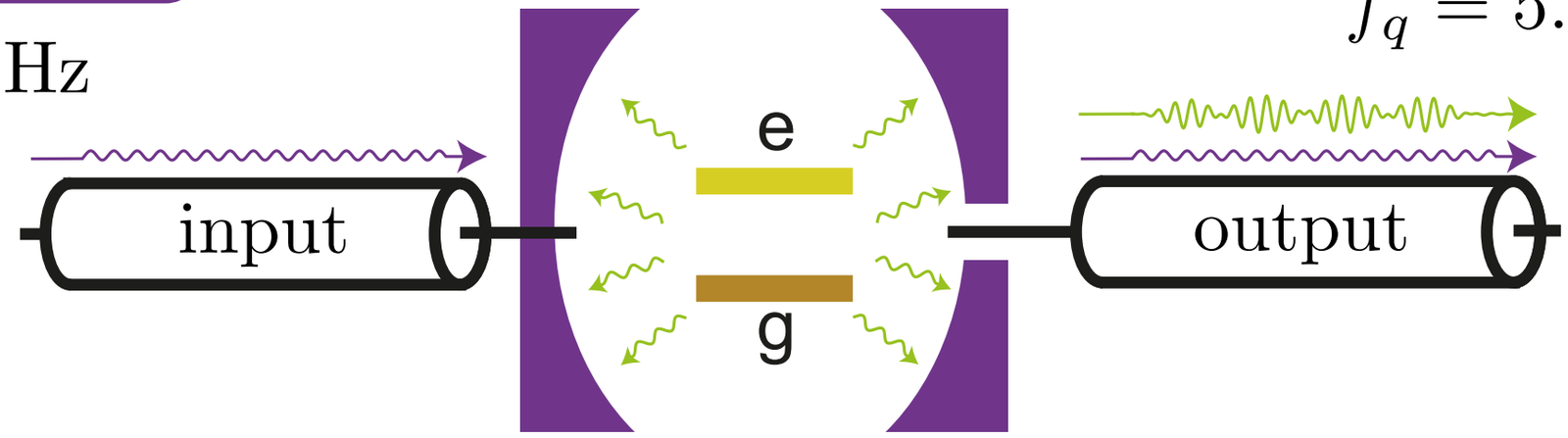


$$H_c = hf_c(a^\dagger a + \frac{1}{2})$$

$$f_c = 7.8 \text{ GHz}$$

$$H_q = hf_q \frac{\sigma_z}{2}$$

$$f_q = 5.4 \text{ GHz}$$

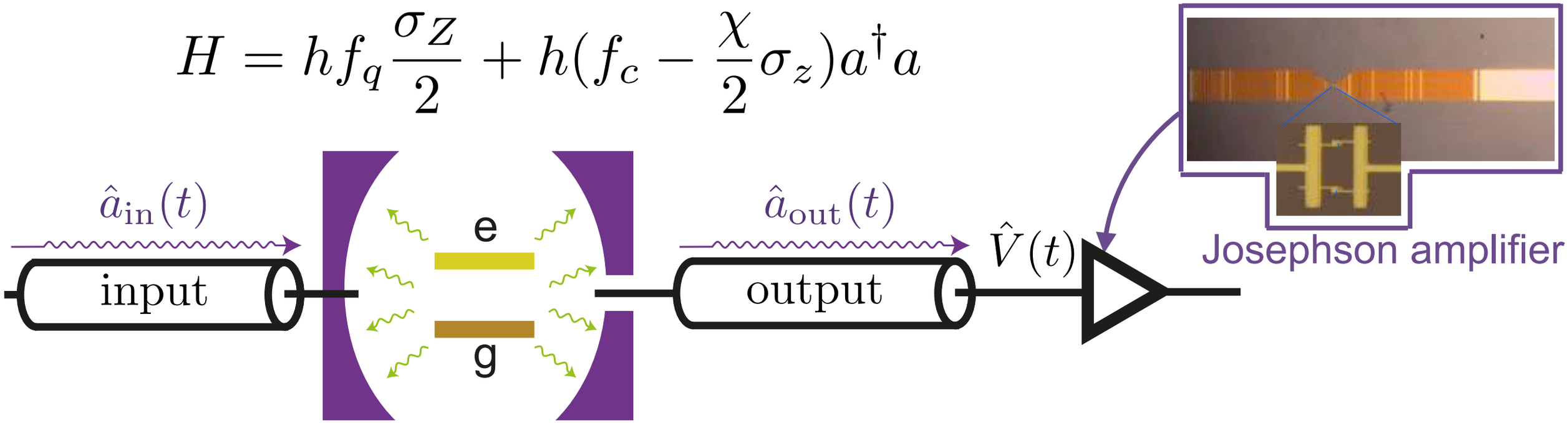


Dispersive Hamiltonian

$$H = hf_c(a^\dagger a + \frac{1}{2}) - h\frac{\chi}{2}\sigma_Z a^\dagger a + hf_q \frac{\sigma_Z}{2}$$

Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



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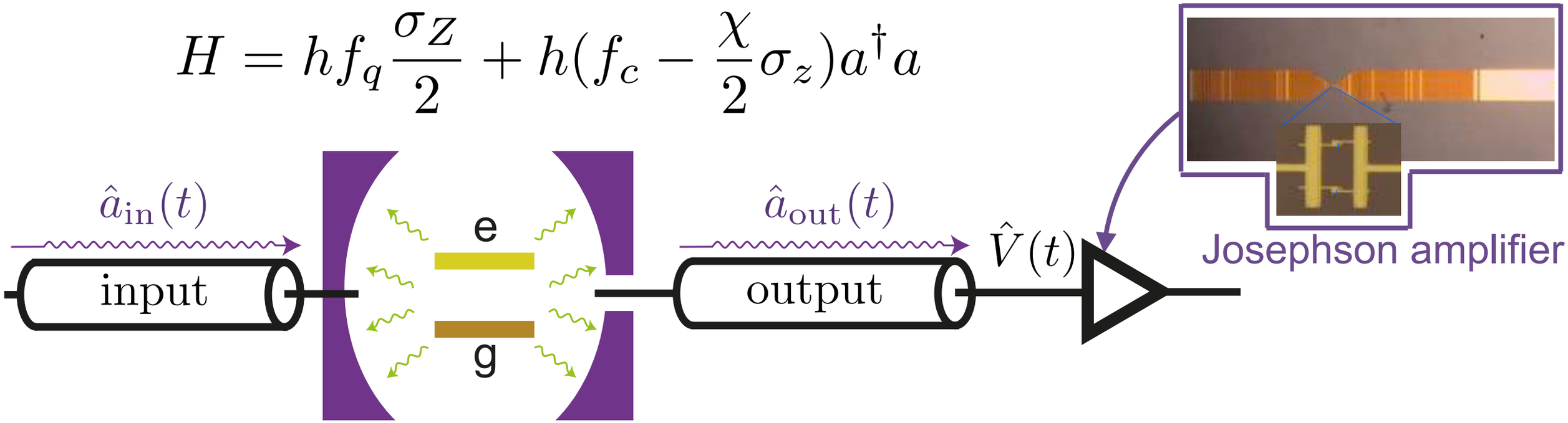

Classically $V(t) = I(t)\cos(2\pi f_c t) + Q(t)\sin(2\pi f_c t)$

$$I_t \to \hat{I}_t \propto \frac{\hat{a}_{\rm out} + \hat{a}^\dagger_{\rm out}}{2} = {\rm Re}(\hat{a}_{\rm out})$$

$$Q_t \to \hat{Q}_t \propto \frac{\hat{a}_{\rm out} - \hat{a}^\dagger_{\rm out}}{2i} = {\rm Im}(\hat{a}_{\rm out})$$

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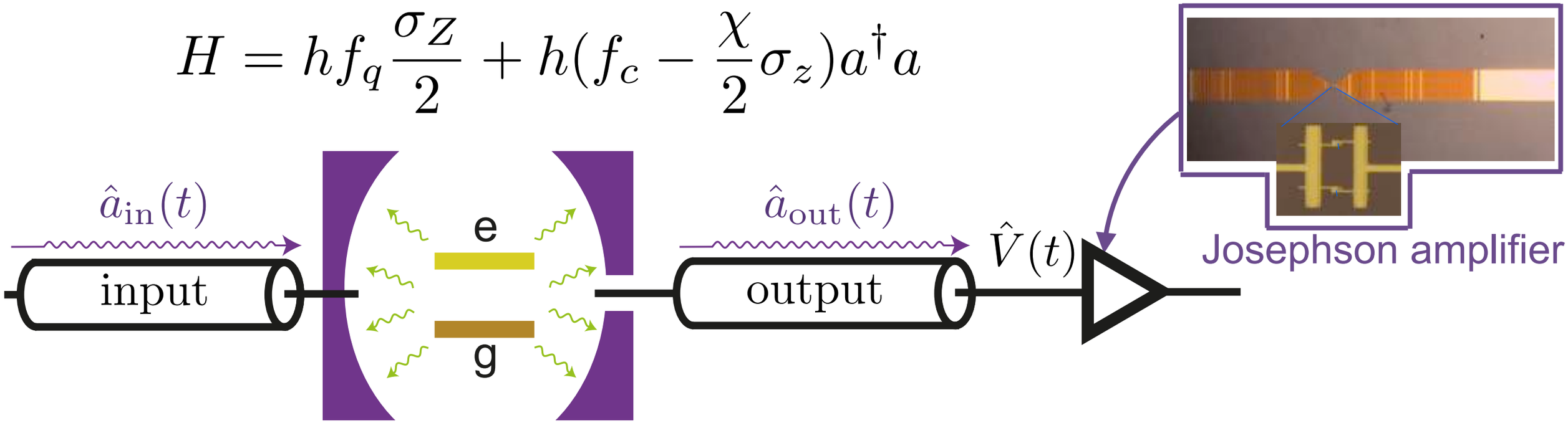

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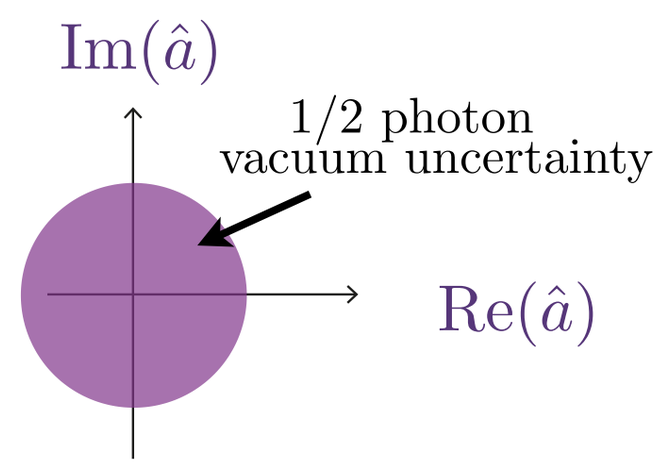
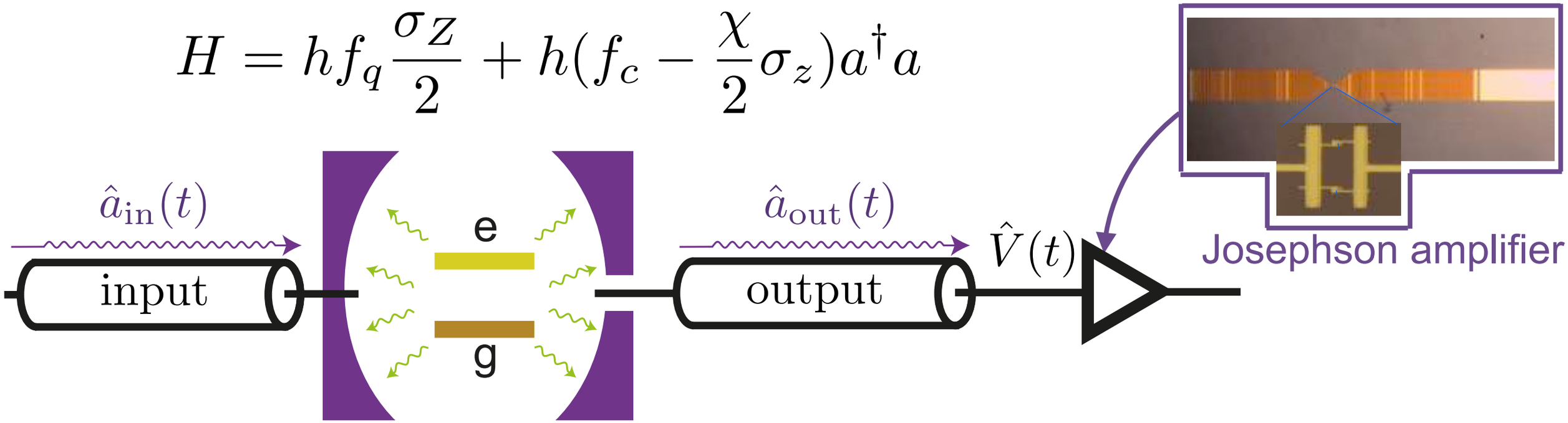


Zero-point fluctuations $|0\rangle$

Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$



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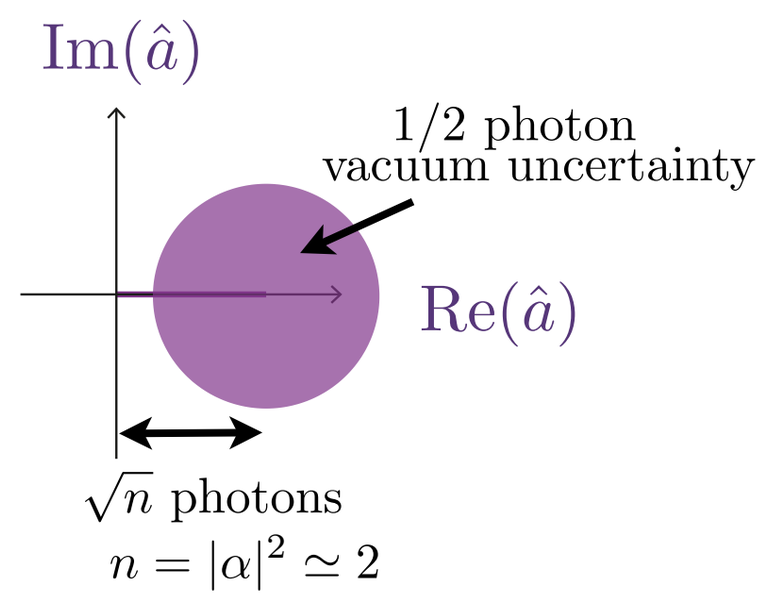
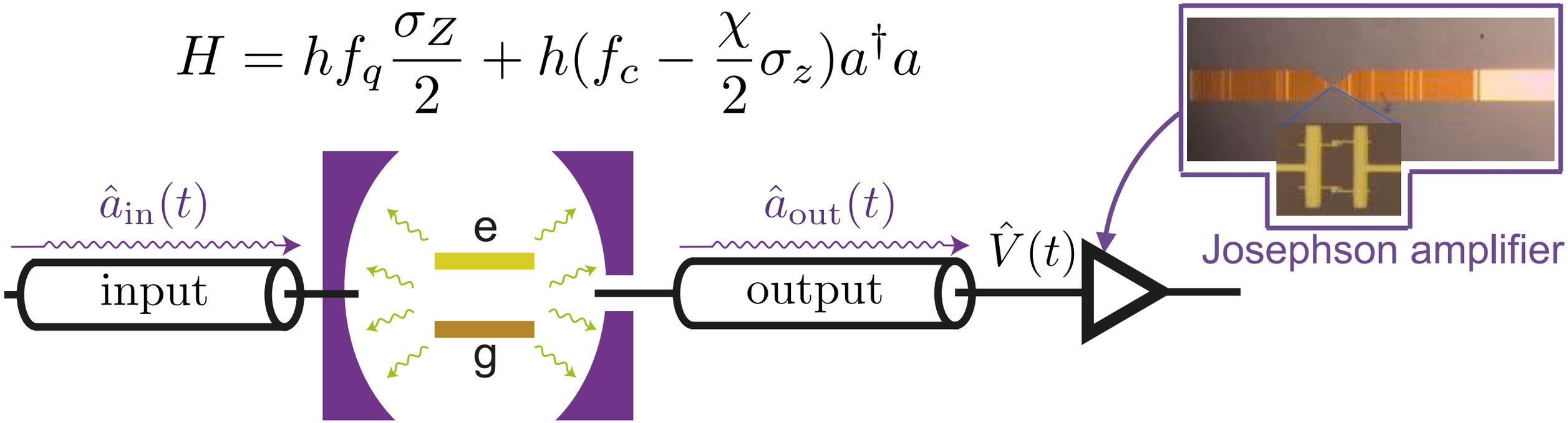
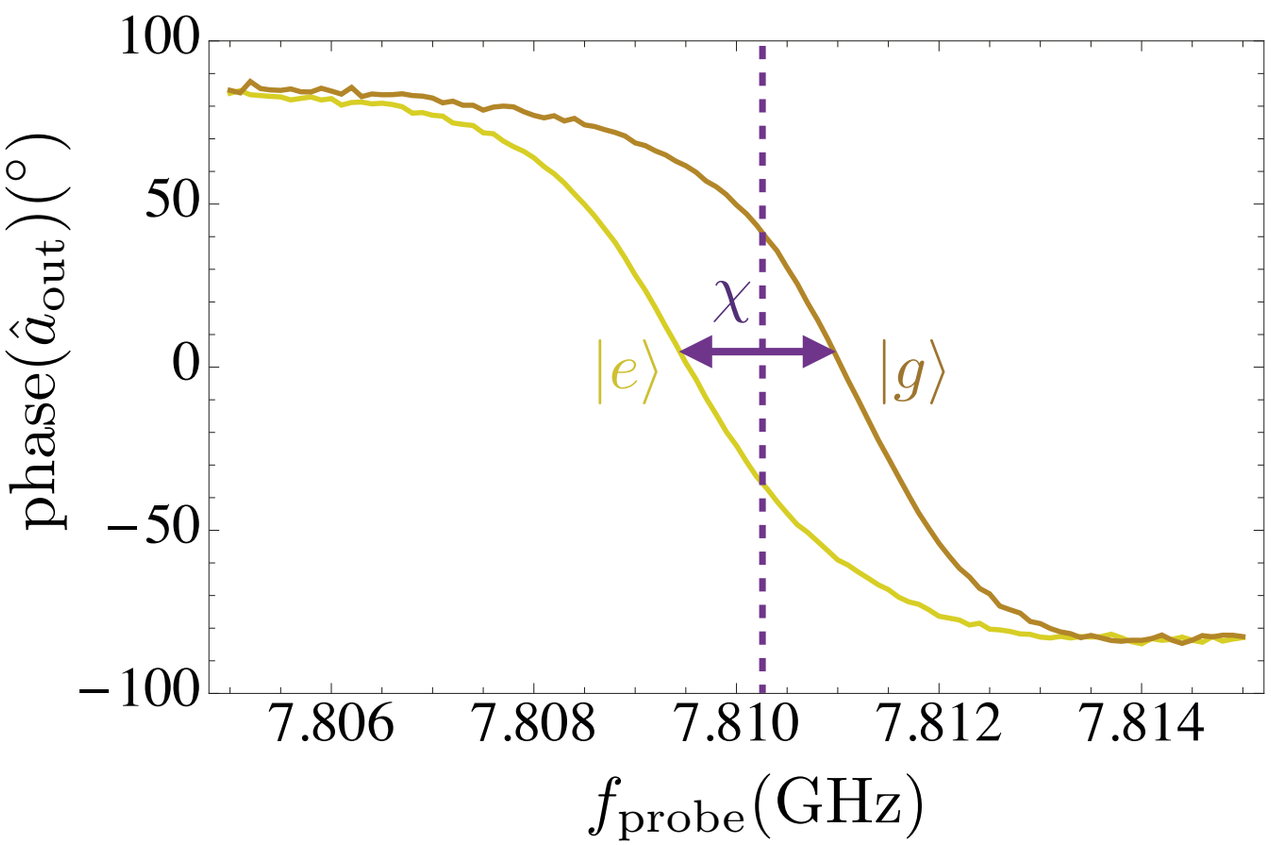


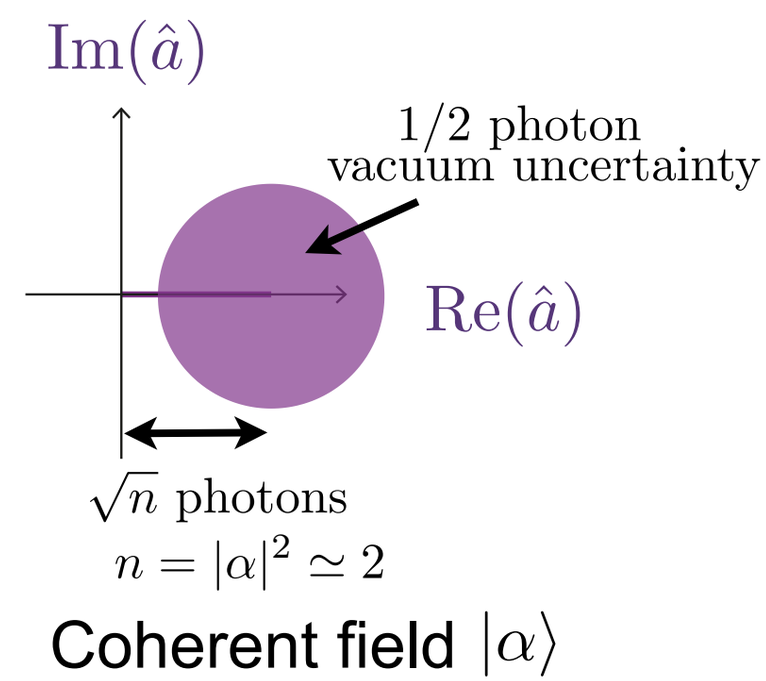
Coherent field $|\alpha\rangle$

Dispersive Measurement

$$H = hf_q \frac{\sigma_Z}{2} + h(f_c - \frac{\chi}{2}\sigma_z)a^\dagger a$$







Coherent field $|\alpha\rangle$

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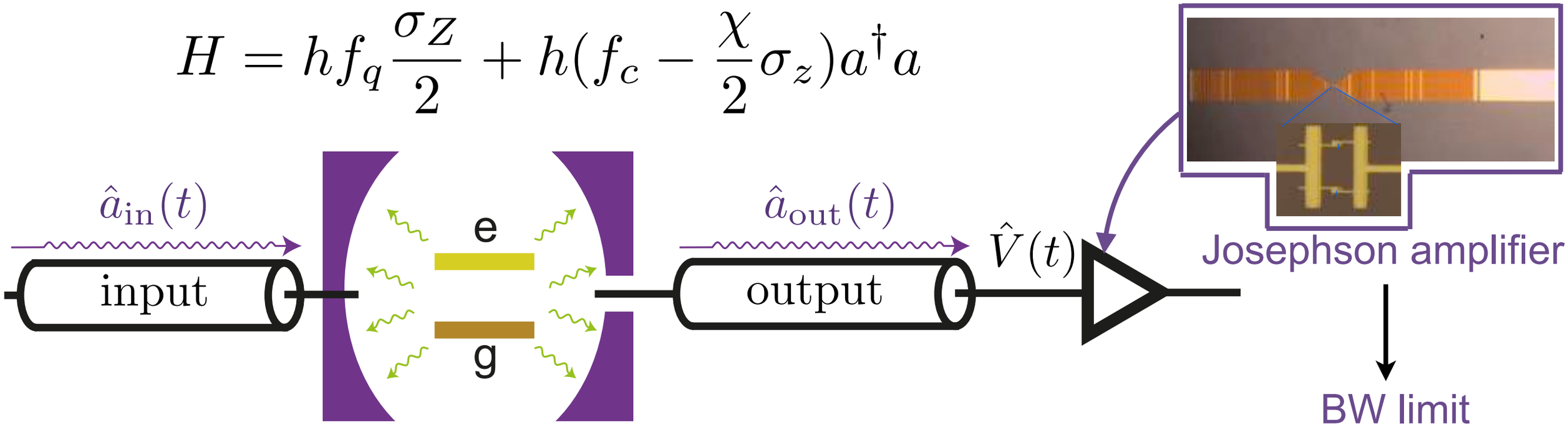
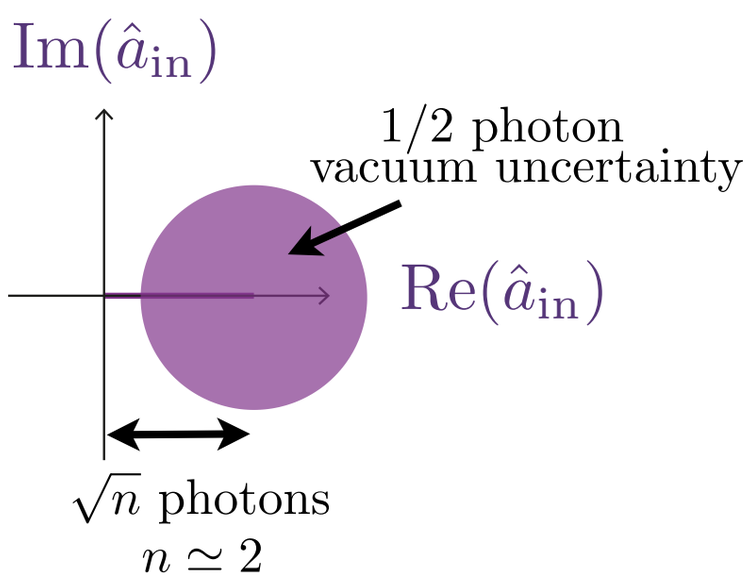
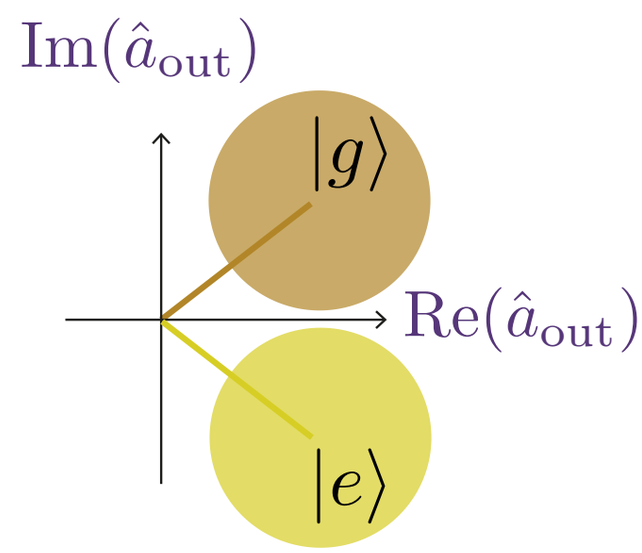
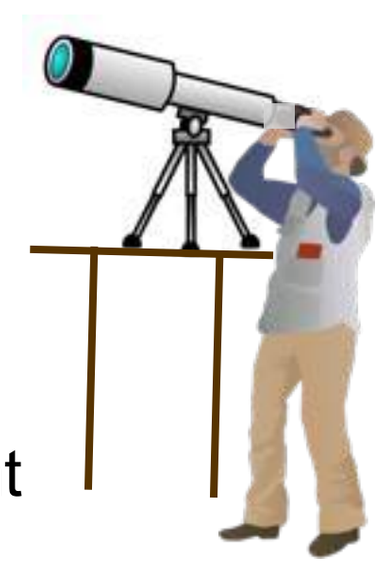


field going in ...



... field coming out

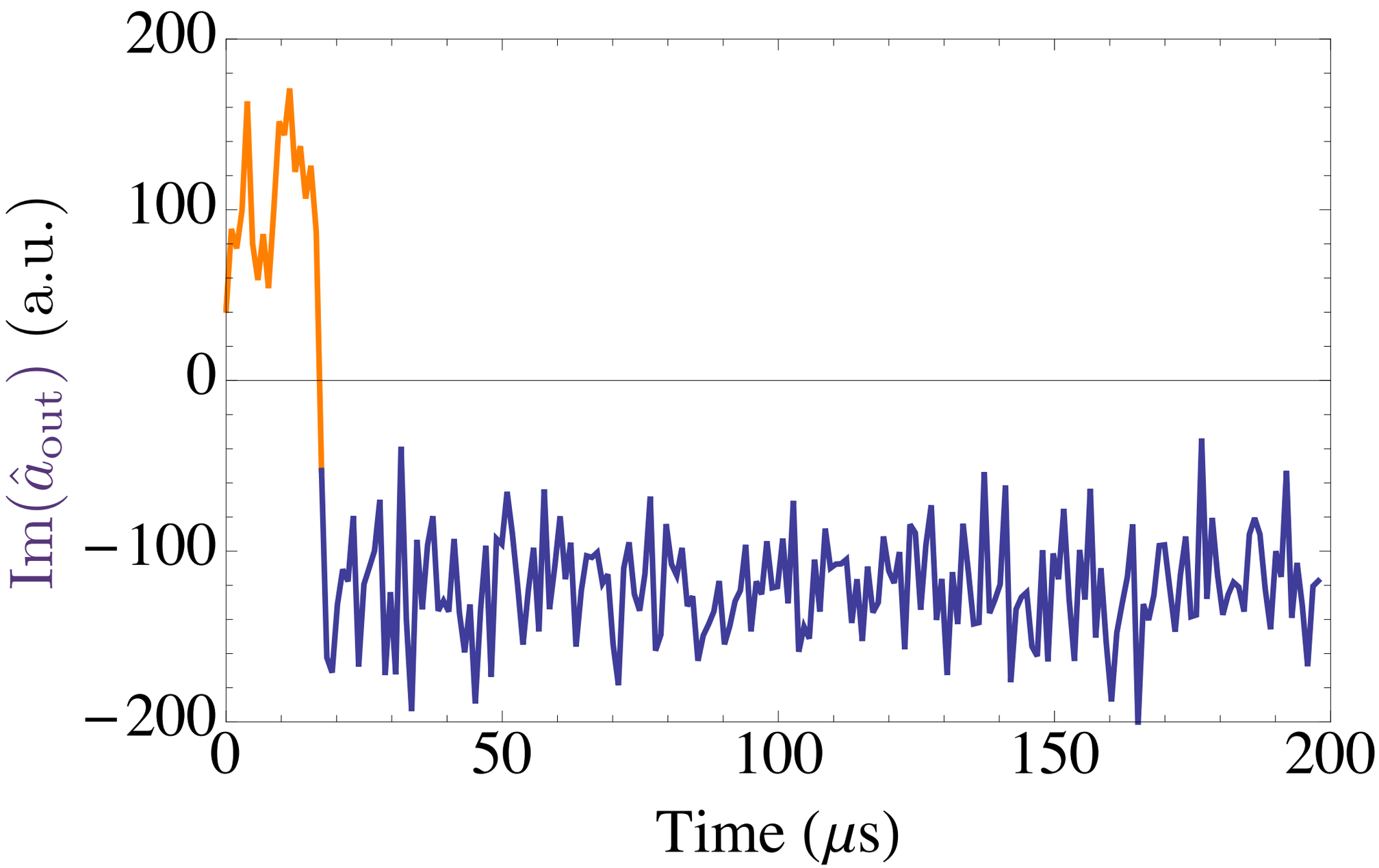




measuring $\mathrm{Im}(\hat{a}_{\rm out})$ ⟶ Strong QND measurement

Quantum jumps

prepare $|e\rangle$ and continuous measurement at 1.8 photons



similar to [Vijay et al., PRL 2011 (Berkeley)]

Quantum jumps

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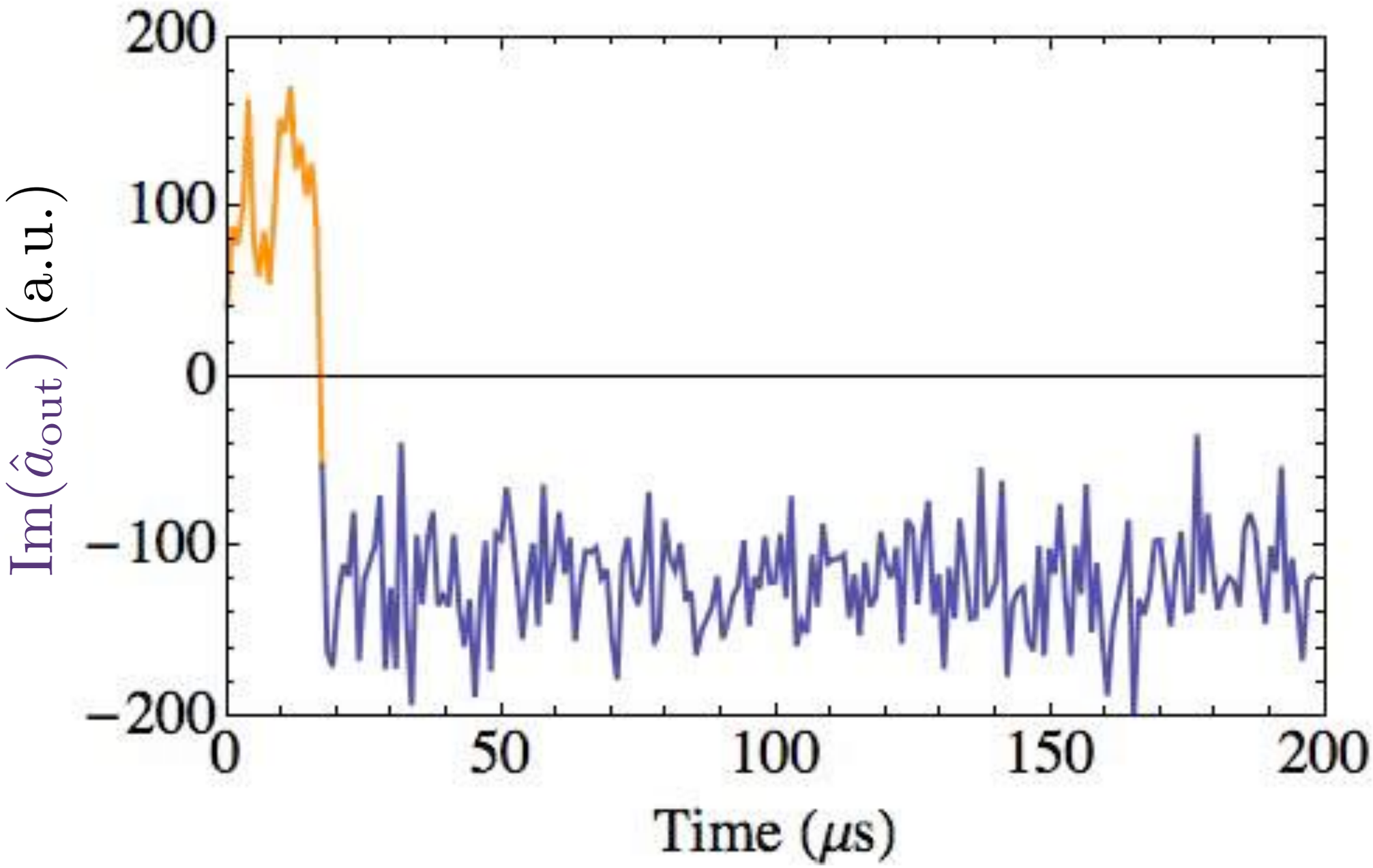

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Quantum jumps

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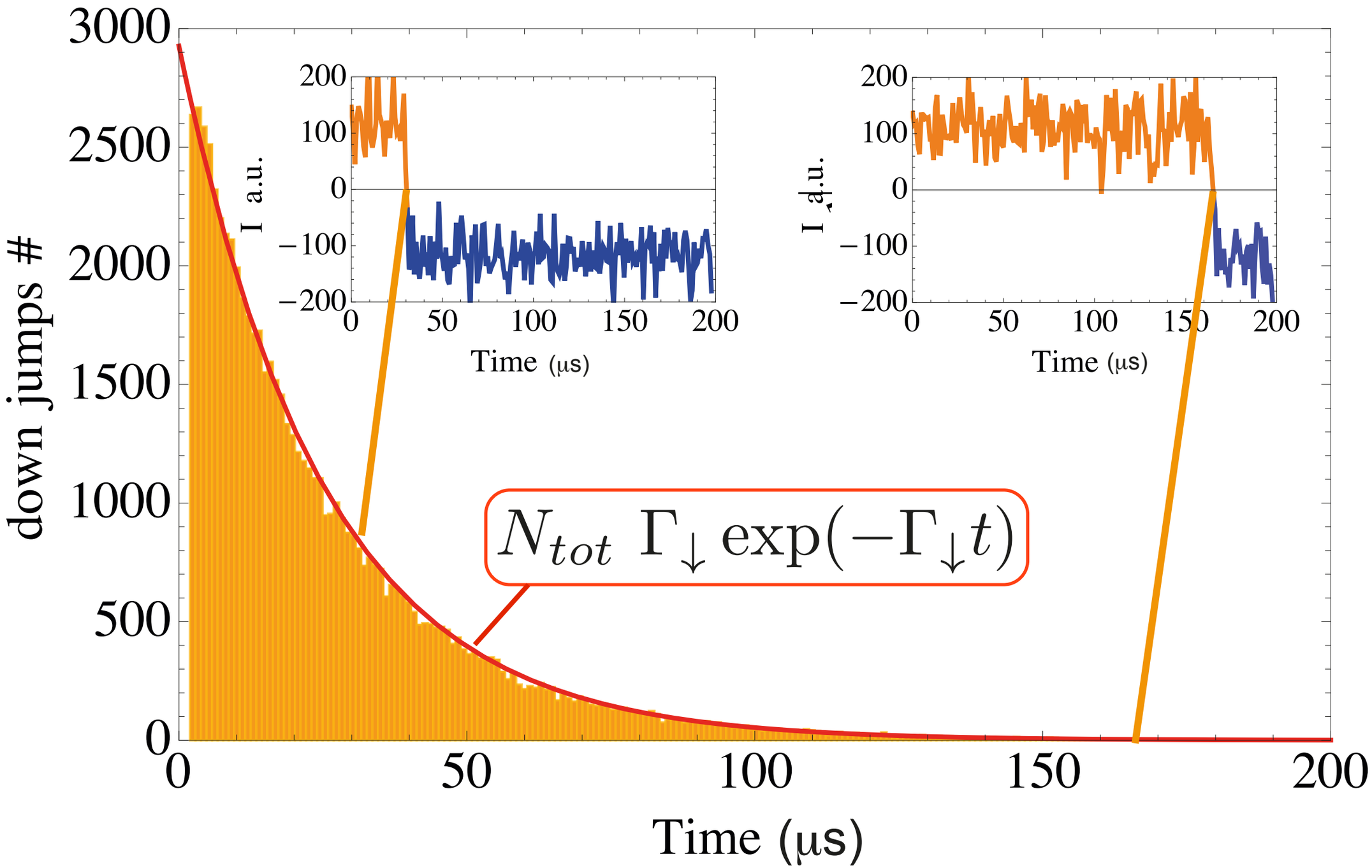
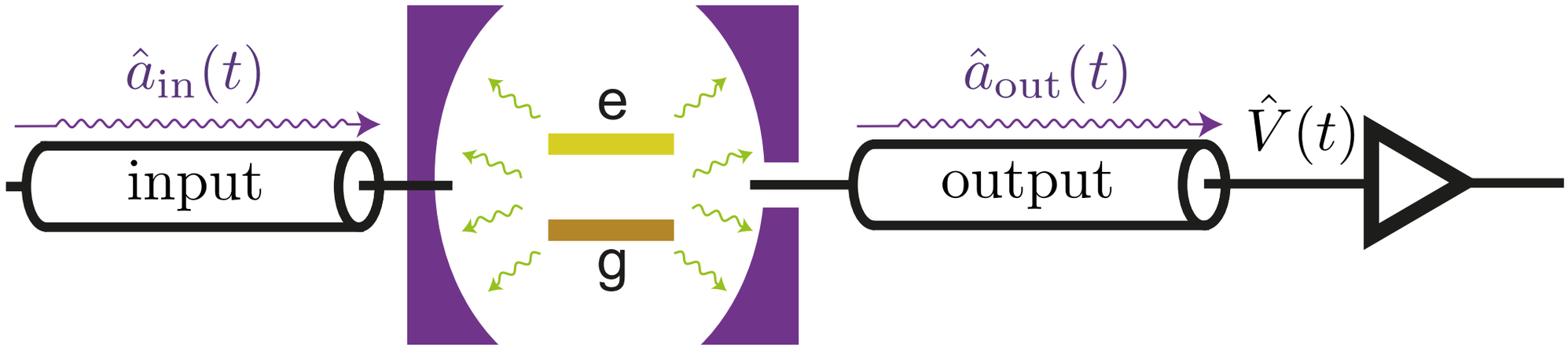


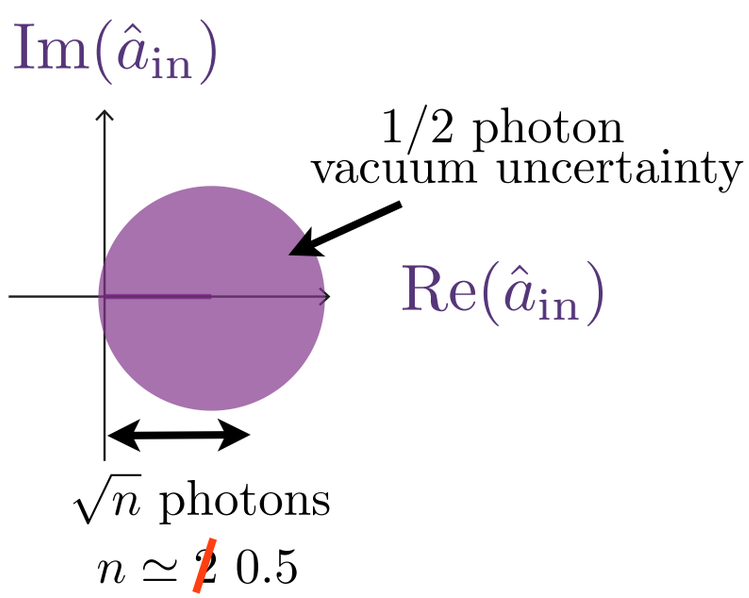
$$\frac{1}{\Gamma_{\downarrow}} \simeq T_1 = 26\ \mu s$$

[Campagne-Ibarcq et al., PRX 2013 (ENS Paris)]

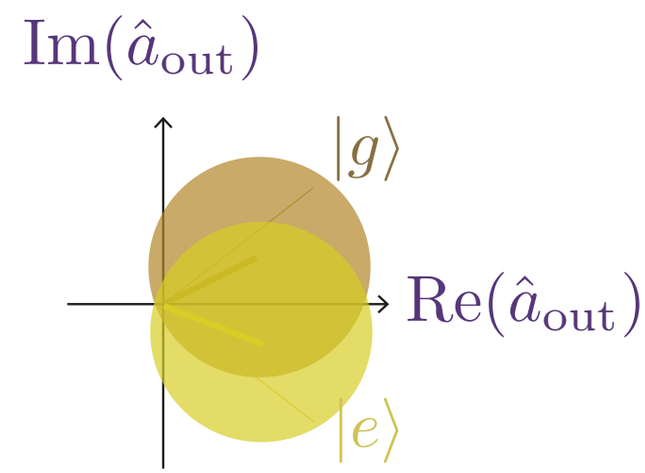
Weak measurement



field going in ...



... field coming out



measuring $\mathrm{Im}(\hat{a}_{\rm out})$ ⟶ Weak QND measurement

Quantum Trajectories - SME

Stochastic Master Equation for a continuous and weak measurement

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^{m} \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^{m} \sqrt{\eta_i}\mathcal{M}_i(\rho_t)dW_{t,i}$$

Dissipation $\mathcal{D}_i(\rho_t) = L_i \rho_t L_i^\dagger - \frac{1}{2}\rho_t L_i^\dagger L_i - \frac{1}{2} L_i^\dagger L_i \rho_t$

Innovation $\mathcal{M}_i(\rho_t) = L_i \rho_t + \rho_t L_i^\dagger - \mathrm{Tr}(L_i \rho_t + \rho_t L_i^\dagger)\rho_t$

Measurement records $dy_t^i = \sqrt{\eta_i}\mathrm{Tr}(L_i \rho_t + \rho_t L_i^\dagger)dt + dW_{t,i}$

Wiener Process

$\mathbb{E}(dW_{t,i}) = 0$

$\mathbb{E}({dW_{t,i}}^2) = dt$

$L_i \rightarrow$ jump operator

$\eta_i \rightarrow$ measurement efficiency

$dt \rightarrow$ limited by amplifier bandwith

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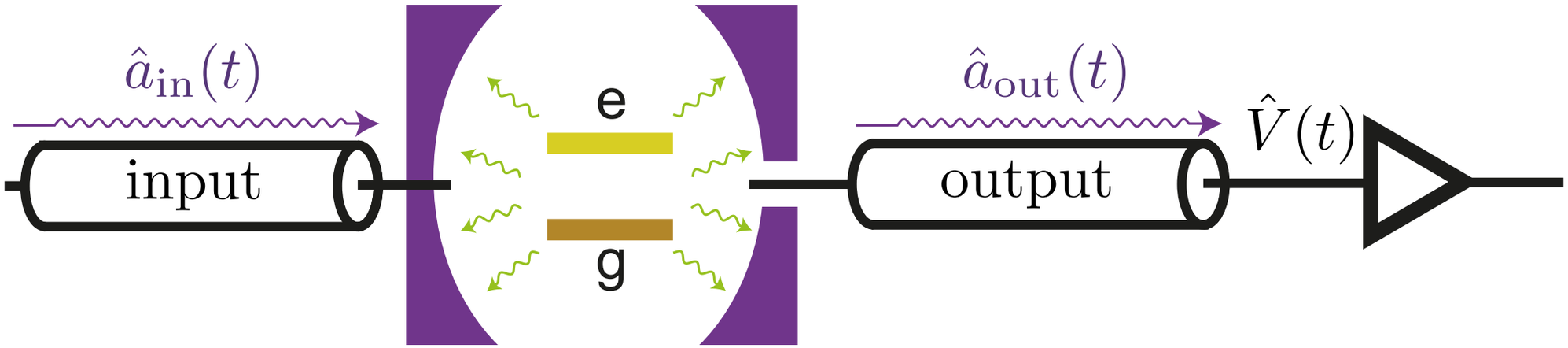
Continuous and weak measurement of z



jump operators

$$L_z = \sqrt{\frac{\Gamma_d}{2}}\sigma_z \qquad \eta_z = 30\%$$

$$L_\downarrow = \sqrt{\Gamma_1}\sigma_- \qquad \eta_\downarrow = 0$$

$$\Gamma_d = 0.47 \text{ MHz}$$

$$\Gamma_1 = 0.57 \text{ MHz}$$

$$d\rho_t = \mathcal{D}_\downarrow(\rho_t)dt + \mathcal{D}_z(\rho_t)dt + \sqrt{\eta_z}\mathcal{M}_z(\rho_t)dW_t$$

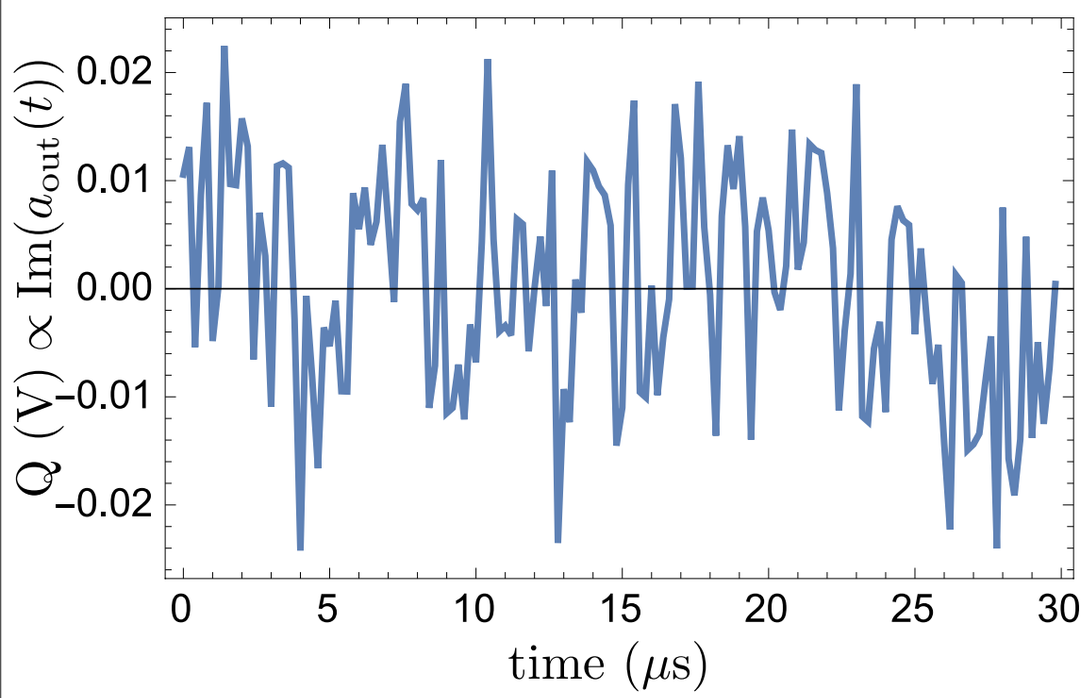
$$dy_t = \sqrt{2\eta_z\Gamma_d}\mathrm{Tr}(\sigma_z\rho)dt + dW_t = \mathrm{Im}(a_{\rm out}(t))dt$$

$$dt = 200 \text{ ns}$$

Continuous and weak measurement of z

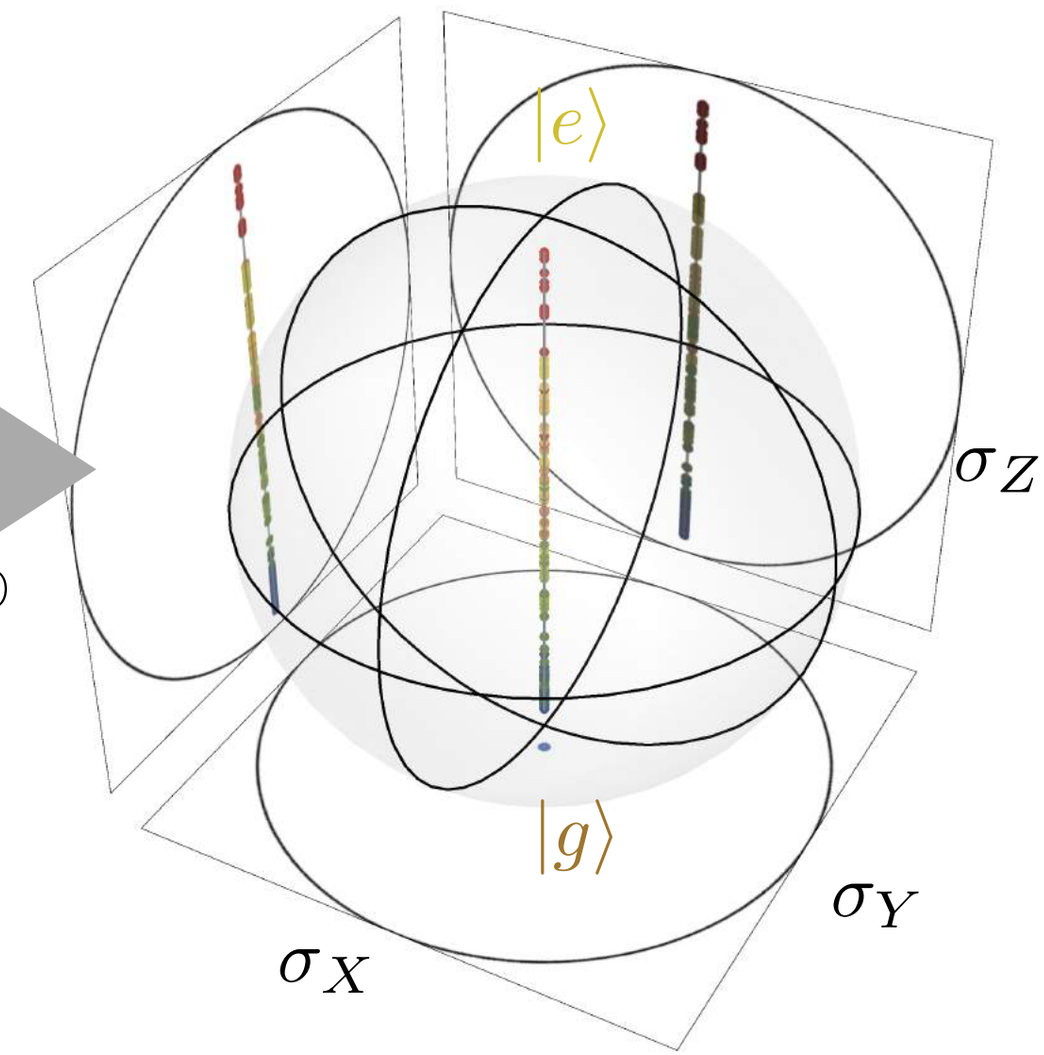
start in $|e\rangle$

$\Gamma_d = 0,47$ MHz



Stochastic Master Eq.

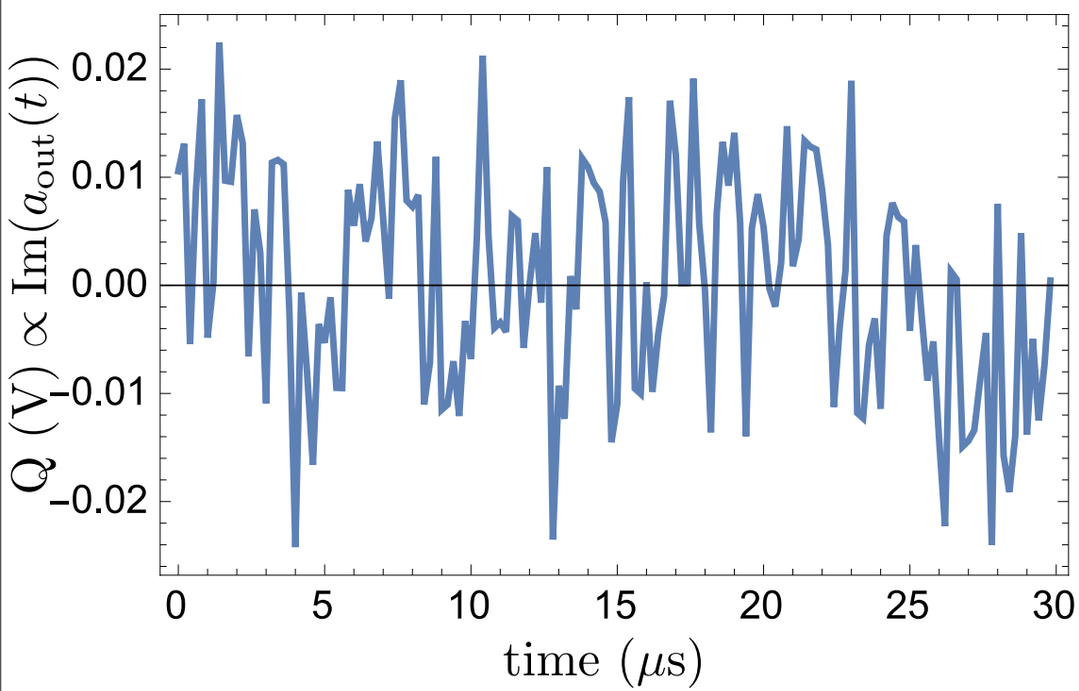
$\eta = 30\ \%$



$$\rho = \frac{\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z}{2}$$

Continuous and weak measurement of z

start in $|e\rangle$



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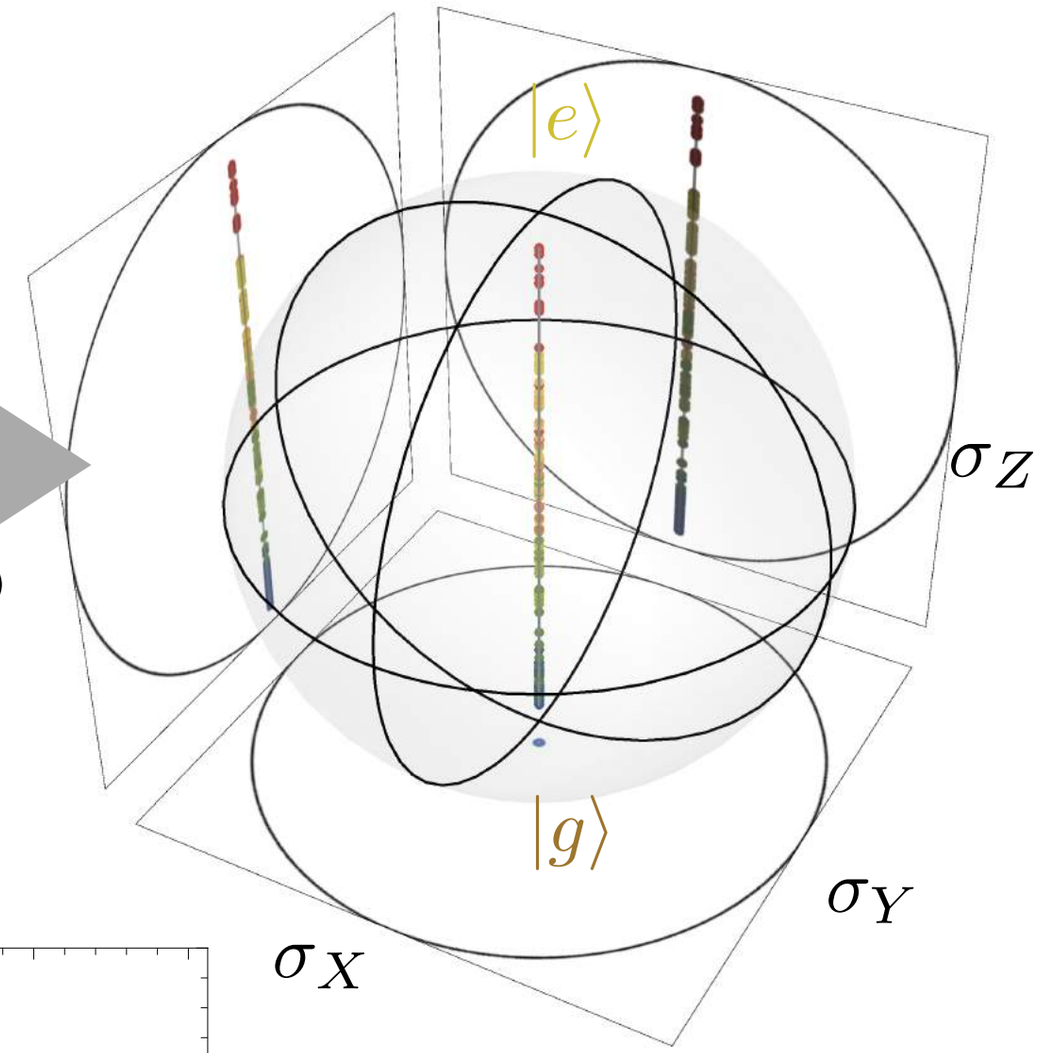
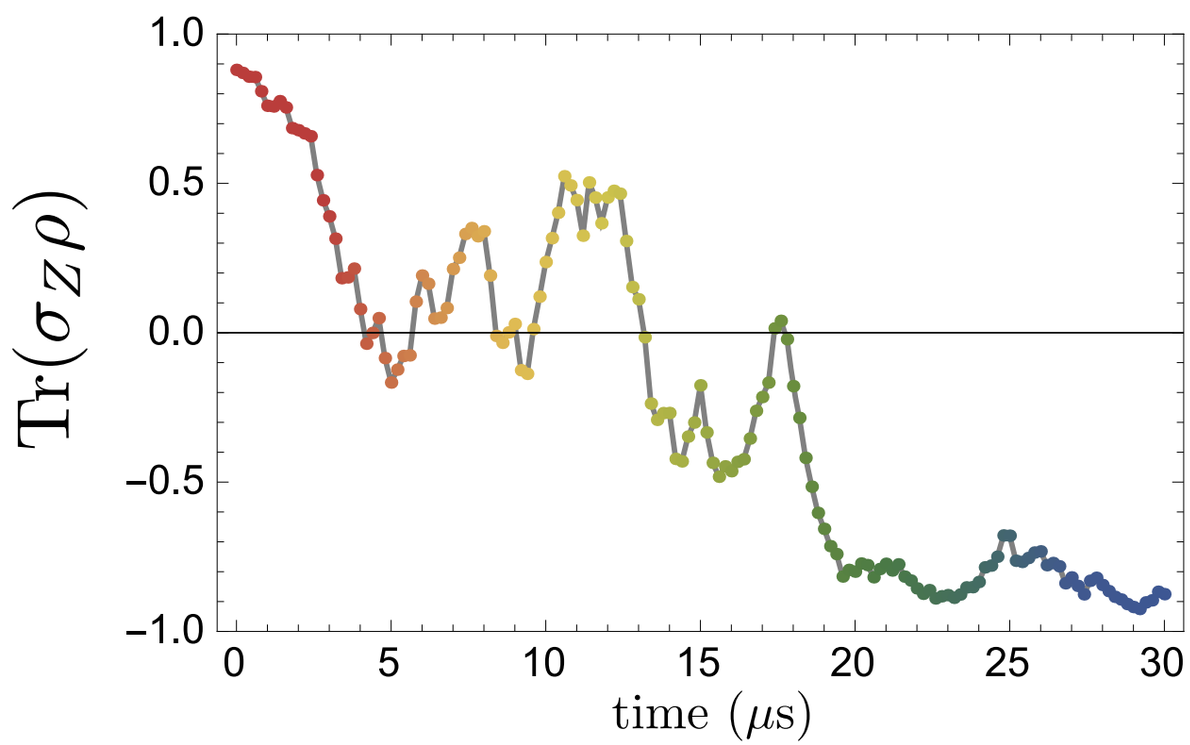


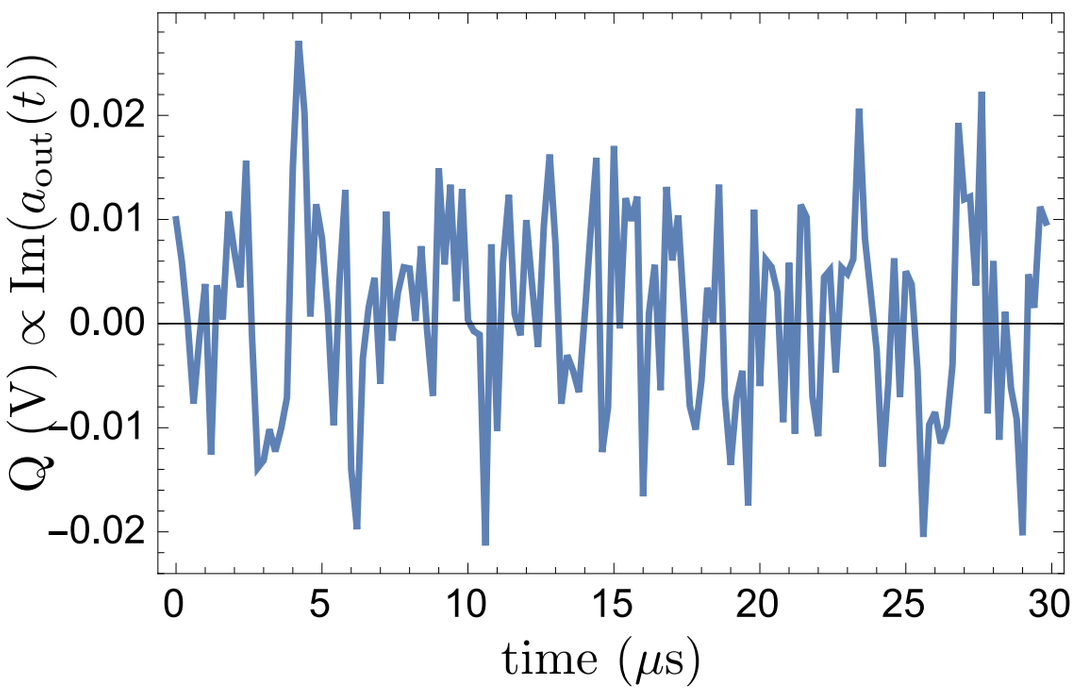


diffusive trajectory

Continuous and weak measurement of z

start in $\frac{|g\rangle + |e\rangle}{\sqrt{2}}$

$\Gamma_d = 0,47$ MHz



Stochastic Master Eq.

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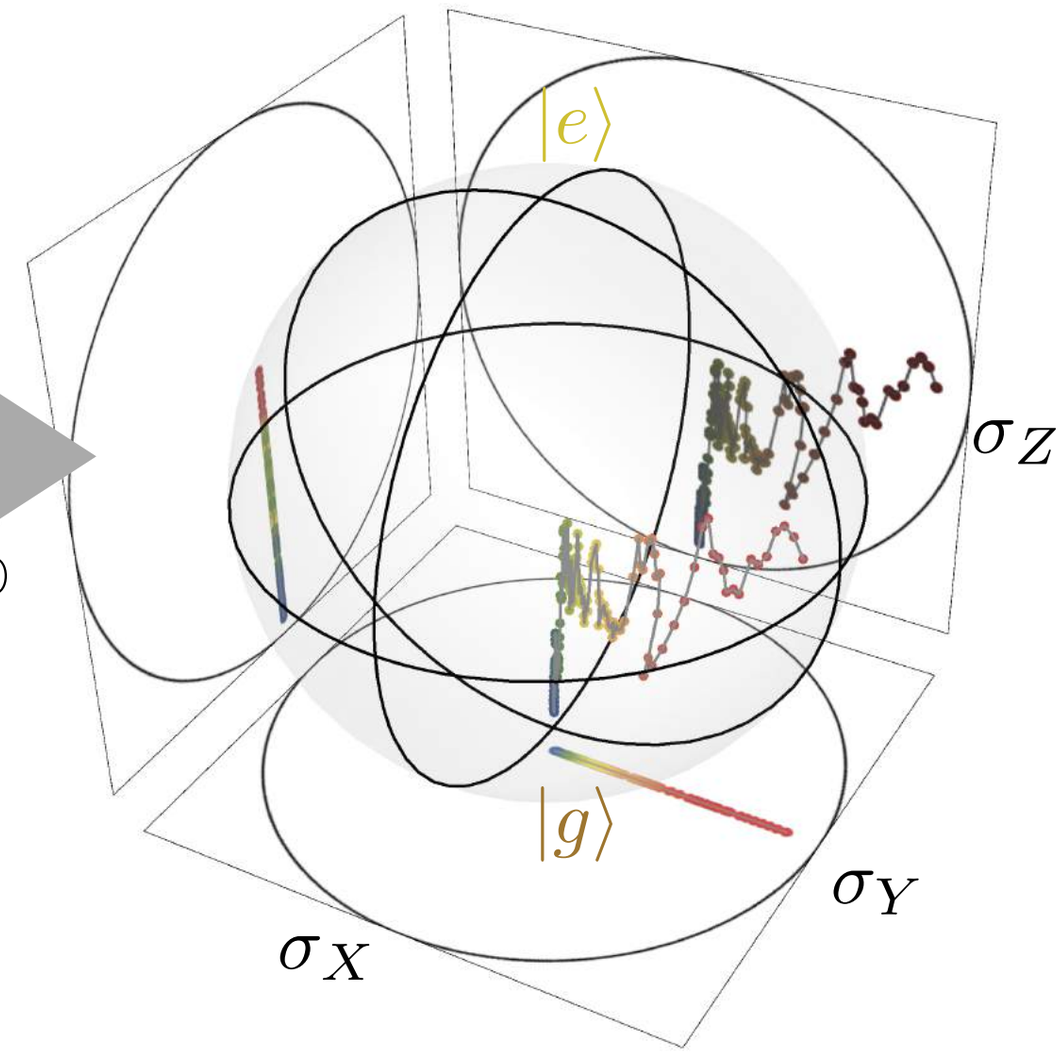
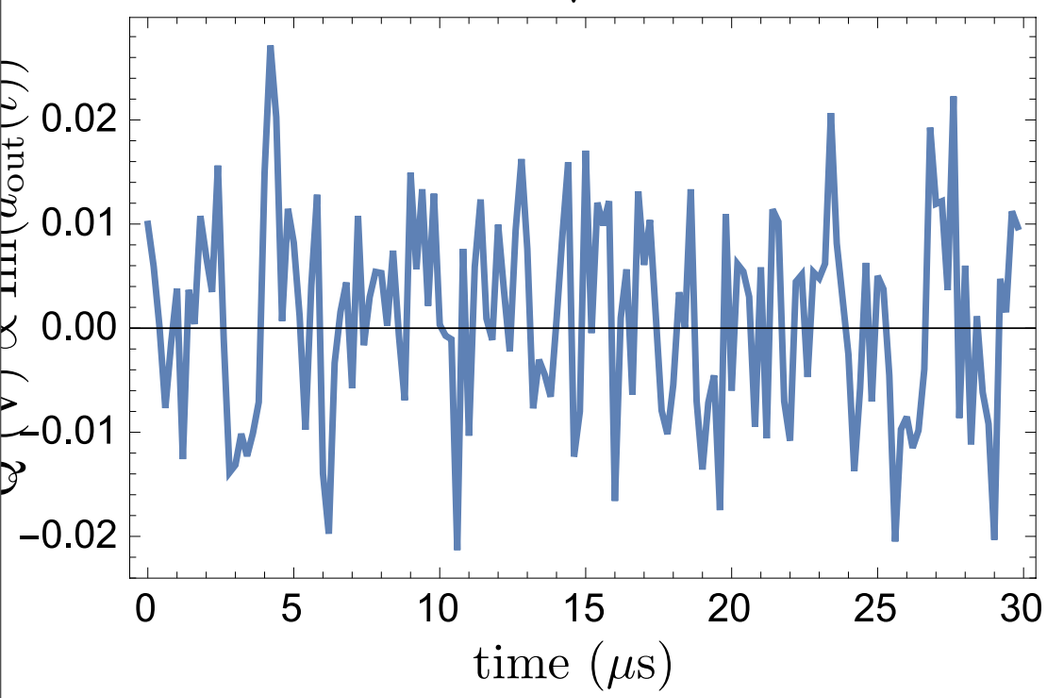


$$\rho = \frac{\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z}{2}$$

Continuous and weak measurement of z

start in $\frac{|g\rangle + |e\rangle}{\sqrt{2}}$



Stochastic Master Eq.

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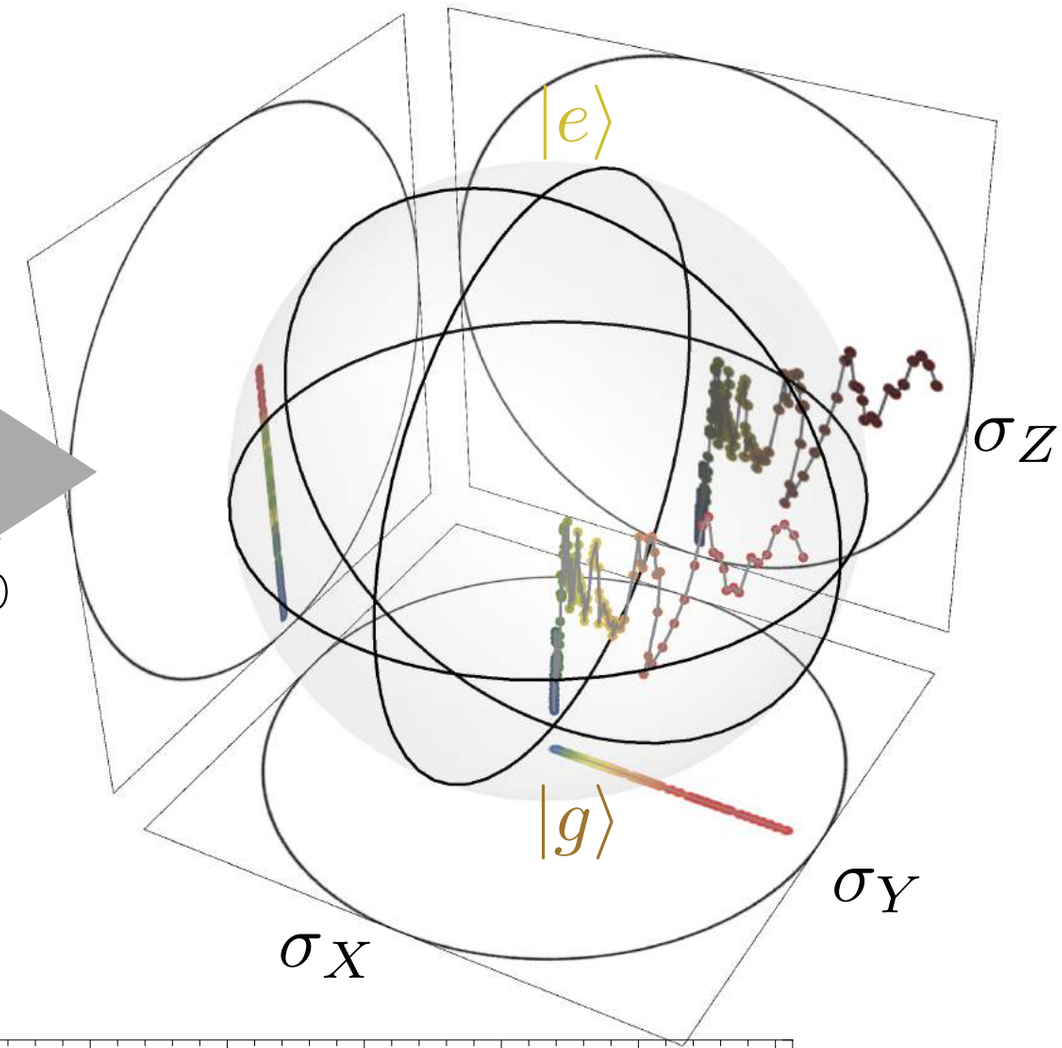
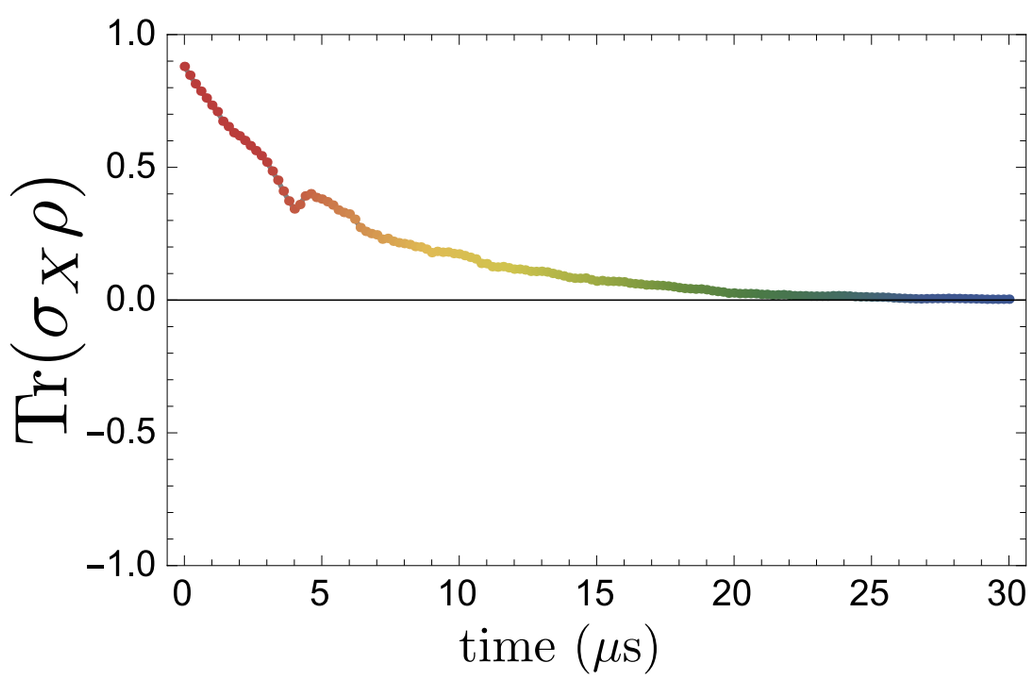
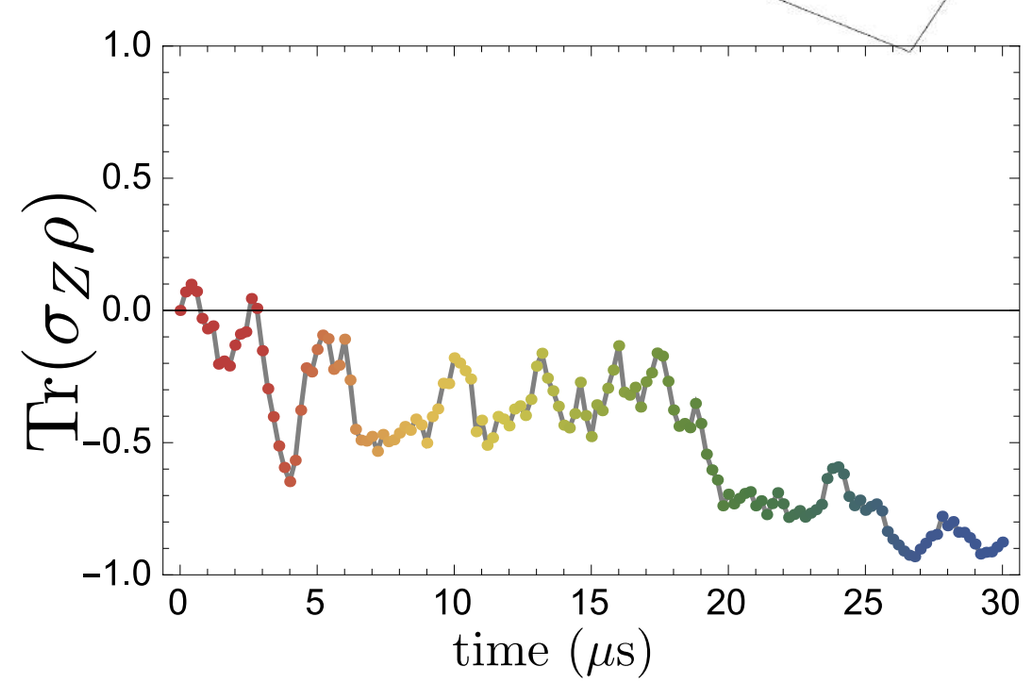
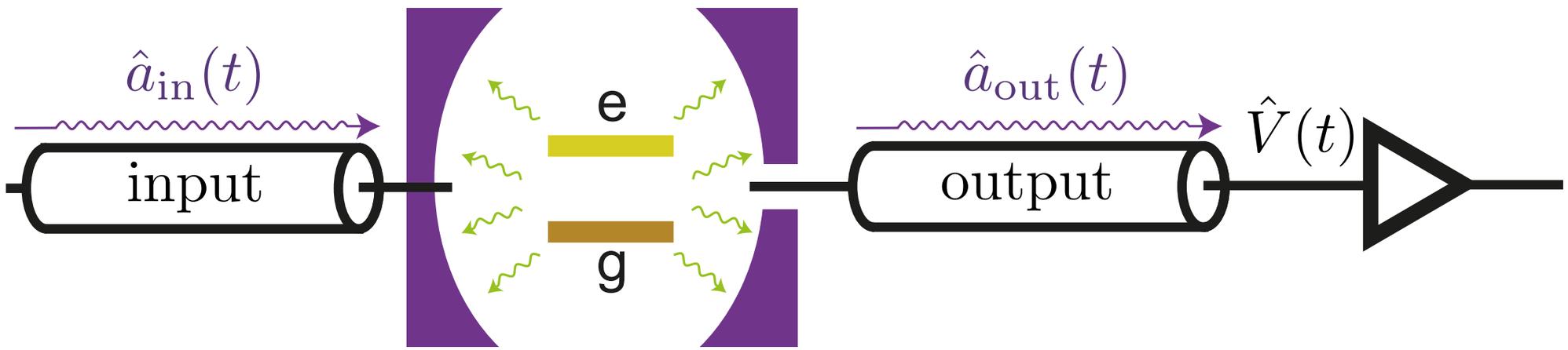






Can we measure fluorescence ?



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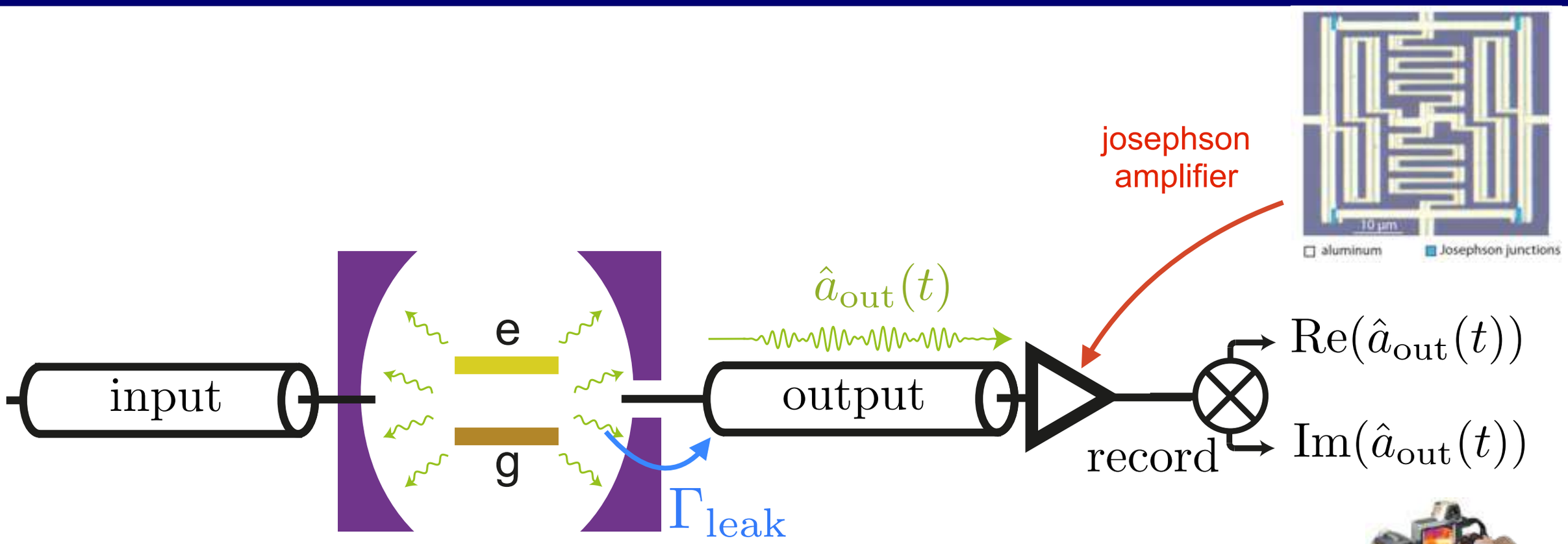
Fluorescence Measurement



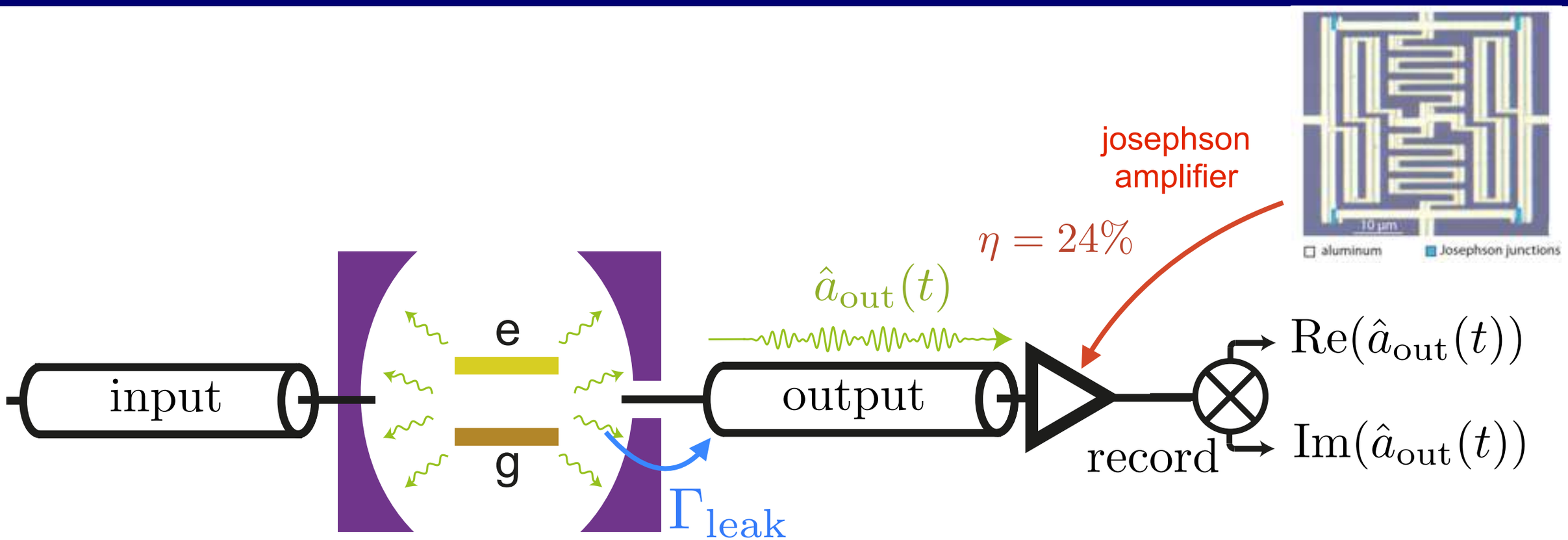
mean signal

$$\langle \hat{a}_{\mathrm{out}} \rangle \propto \sqrt{\Gamma_{\mathrm{leak}}} \langle \sigma_- \rangle = \sqrt{\Gamma_1} \langle \sigma_- \rangle$$

$$\sigma_- = |g\rangle \langle e| = \frac{\sigma_x - i\sigma_y}{2}$$

$$\Gamma_1 = \frac{1}{T_1} = \frac{1}{10\ \mu\mathrm{s}}$$

Fluorescence Measurement



$$L_1 = \sqrt{\frac{\Gamma_1}{2}}\sigma_- \quad L_2 = i\sqrt{\frac{\Gamma_1}{2}}\sigma_-$$

$$dy_{t,1} = \mathrm{Re}(\hat{a}_{\rm out}(t))dt = \sqrt{\eta\Gamma_1/2}\mathrm{Tr}(\sigma_X\rho)dt + dW_{t,1}$$

$$dy_{t,2} = \mathrm{Im}(\hat{a}_{\rm out}(t))dt = \sqrt{\eta\Gamma_1/2}\mathrm{Tr}(\sigma_Y\rho)dt + dW_{t,2}$$

average outcome

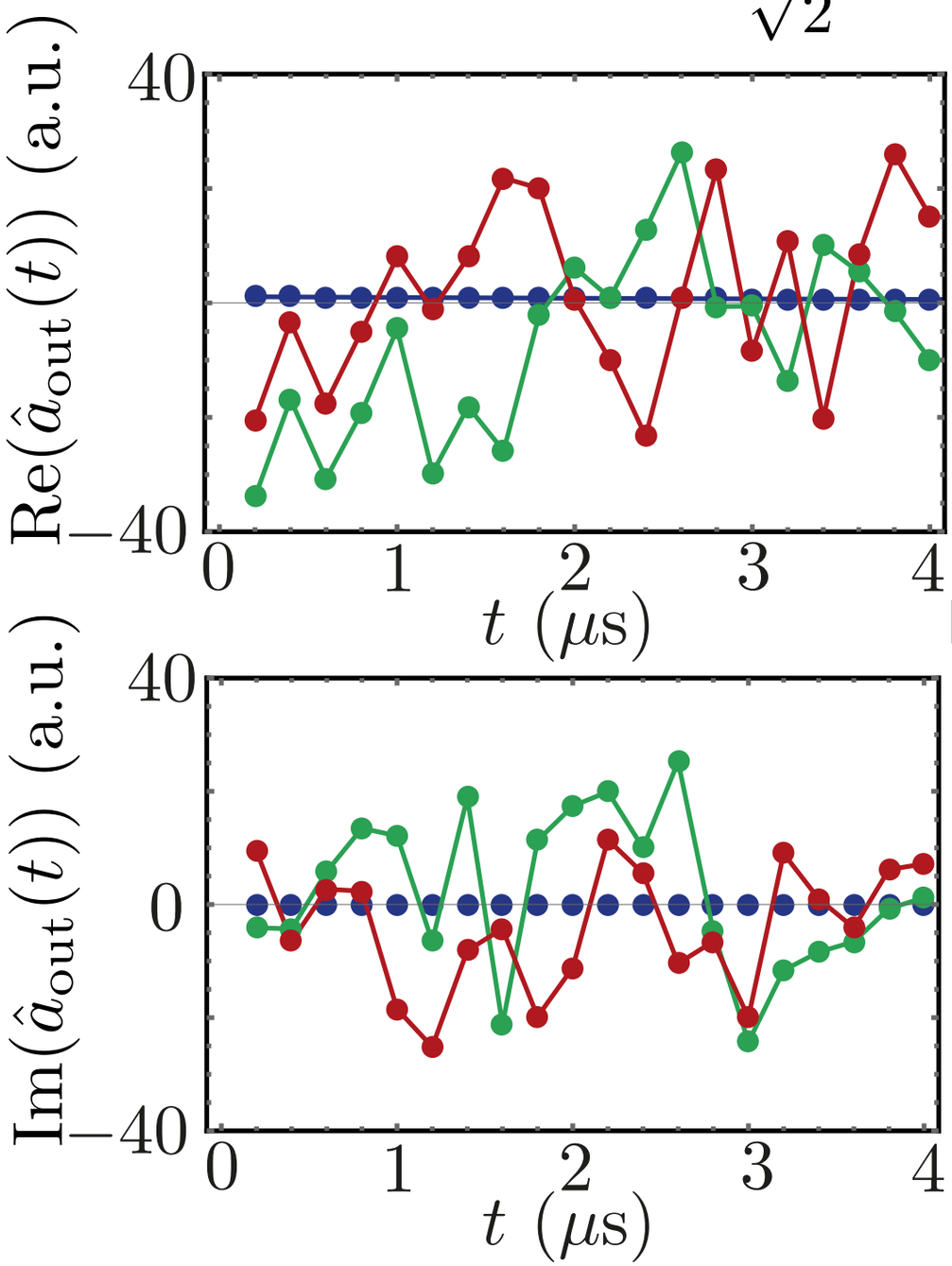
noise
(Wiener)

$$\mathrm{d}\rho_t = \mathcal{D}_1(\rho_t)dt + \mathcal{D}_2(\rho_t)dt + \sqrt{\eta}\mathcal{M}_1(\rho_t)dW_{t,1} + \sqrt{\eta}\mathcal{M}_2(\rho_t)dW_{t,2}$$

Two realizations

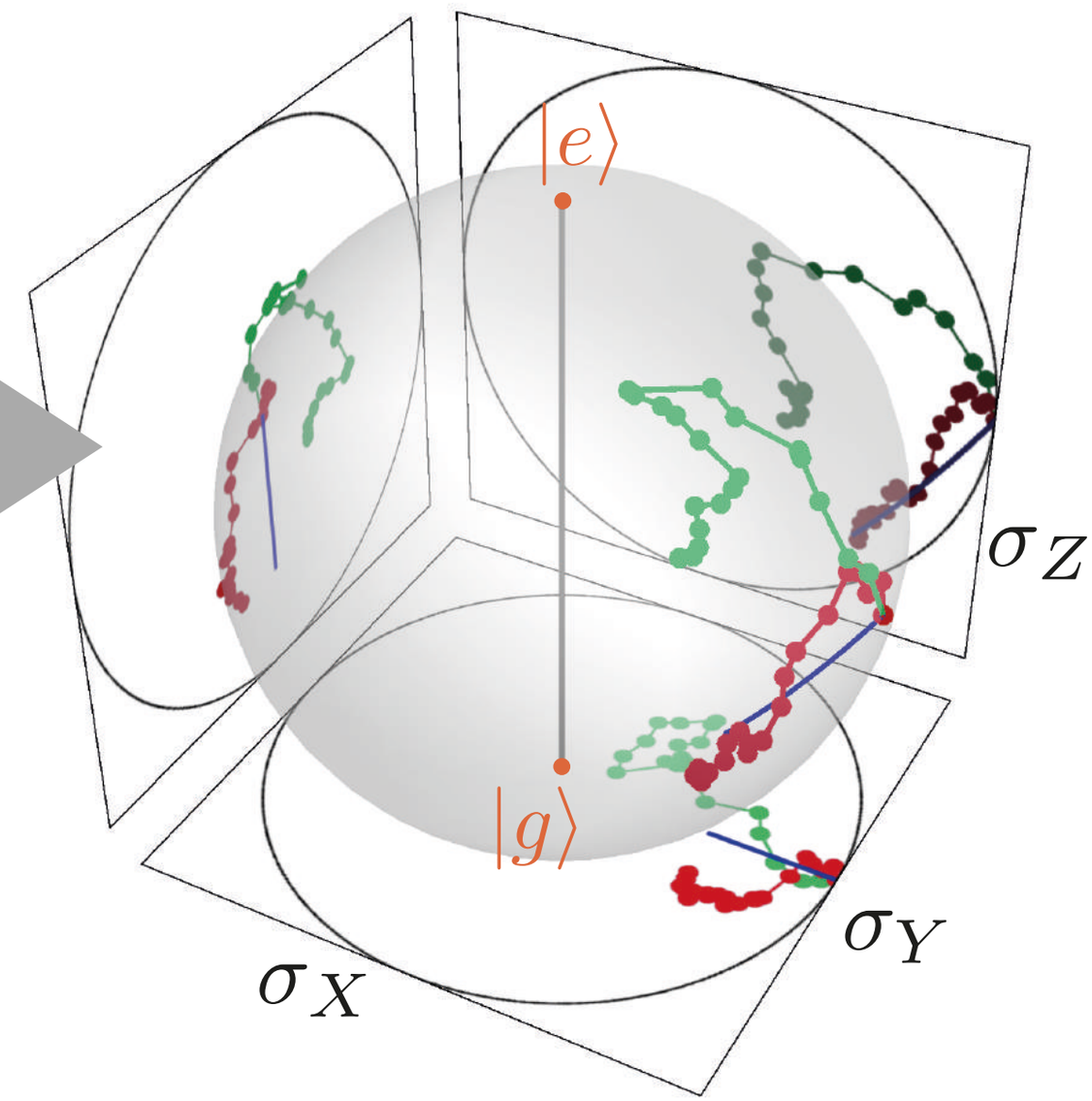
start in $|+x\rangle = \dfrac{|g\rangle + |e\rangle}{\sqrt{2}}$

$\mathrm{d}t = 200\ \mathrm{ns}$



Stochastic Master Eq.

$\eta = 24\ \%$

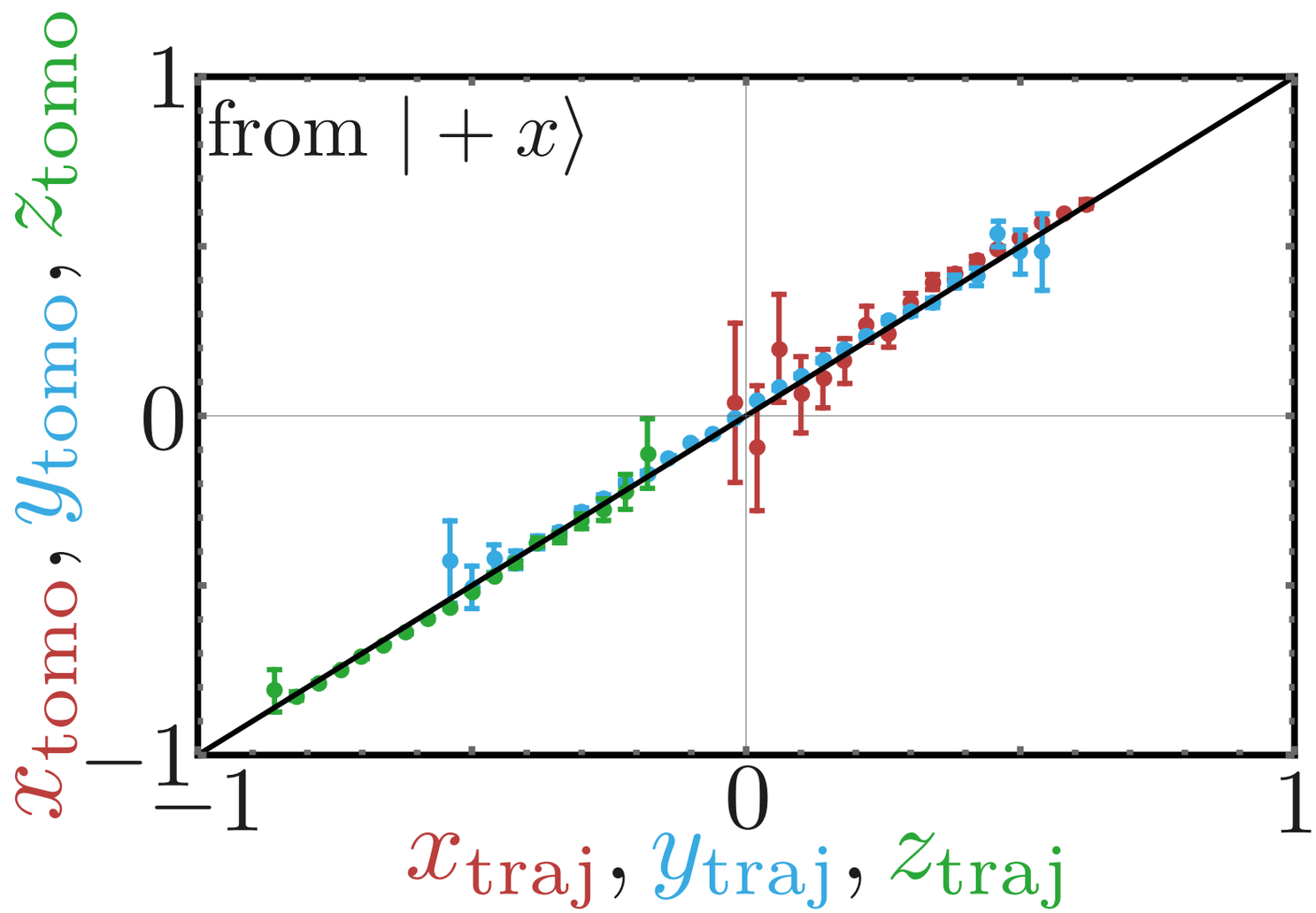


$$\rho = \frac{\mathbf{1} + x\sigma_x + y\sigma_y + z\sigma_z}{2}$$

Reality and interest of Quantum Trajectories

$$\{\rho(T) \text{ close to } \rho_0\} \longrightarrow \overline{\mathrm{Tomo}(\rho(T))} \stackrel{?}{=} \rho_0$$

tomography by strong dispersive measurement



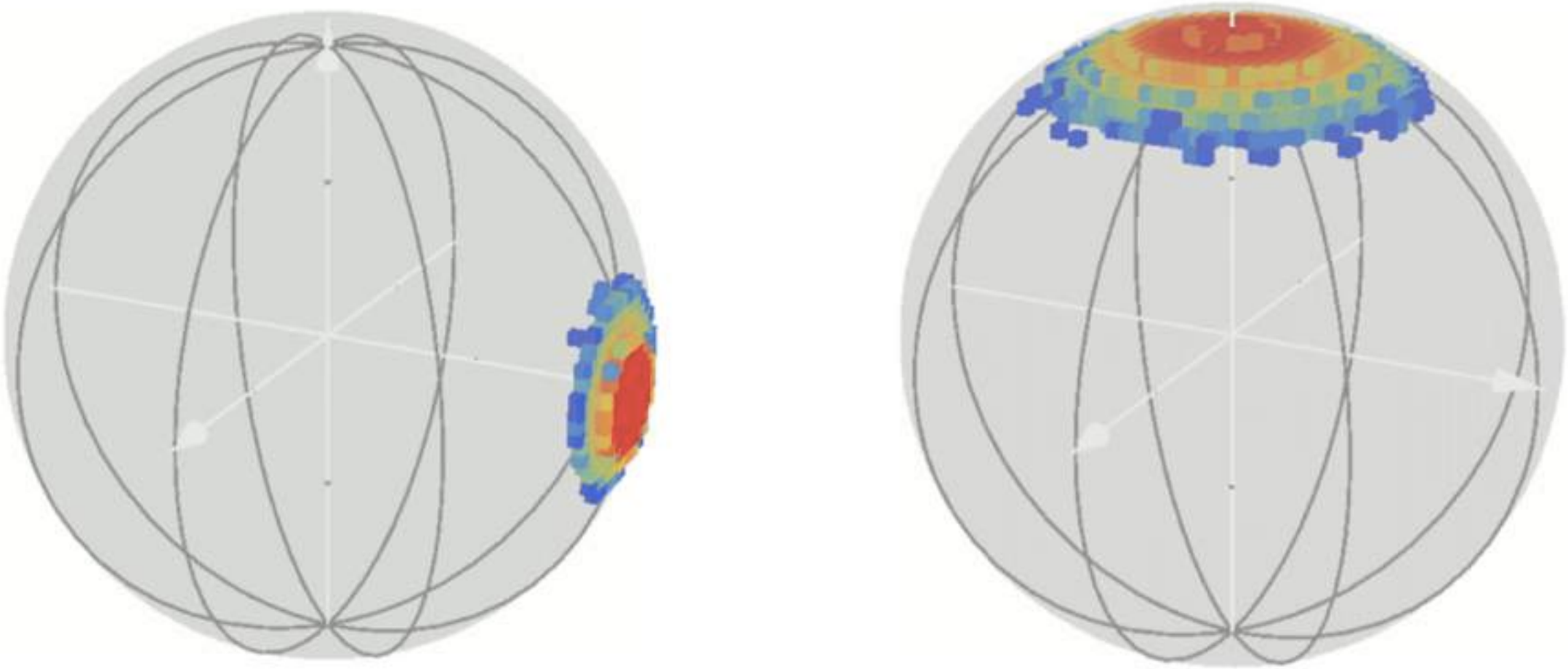
trajectory from fluorescence record

[Campagne-Ibarcq *et al.*, PRX 2016]

Statistics of relaxation trajectories

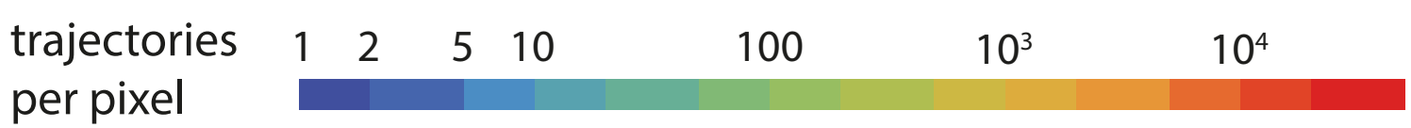
start in $|+x\rangle = \dfrac{|g\rangle + |e\rangle}{\sqrt{2}}$

start in $|e\rangle$



10^6 experiments

$\mathbb{P}(\rho_t)$



[Campagne-Ibarcq *et al.*, PRX 2016]

Statistics of relaxation trajectories

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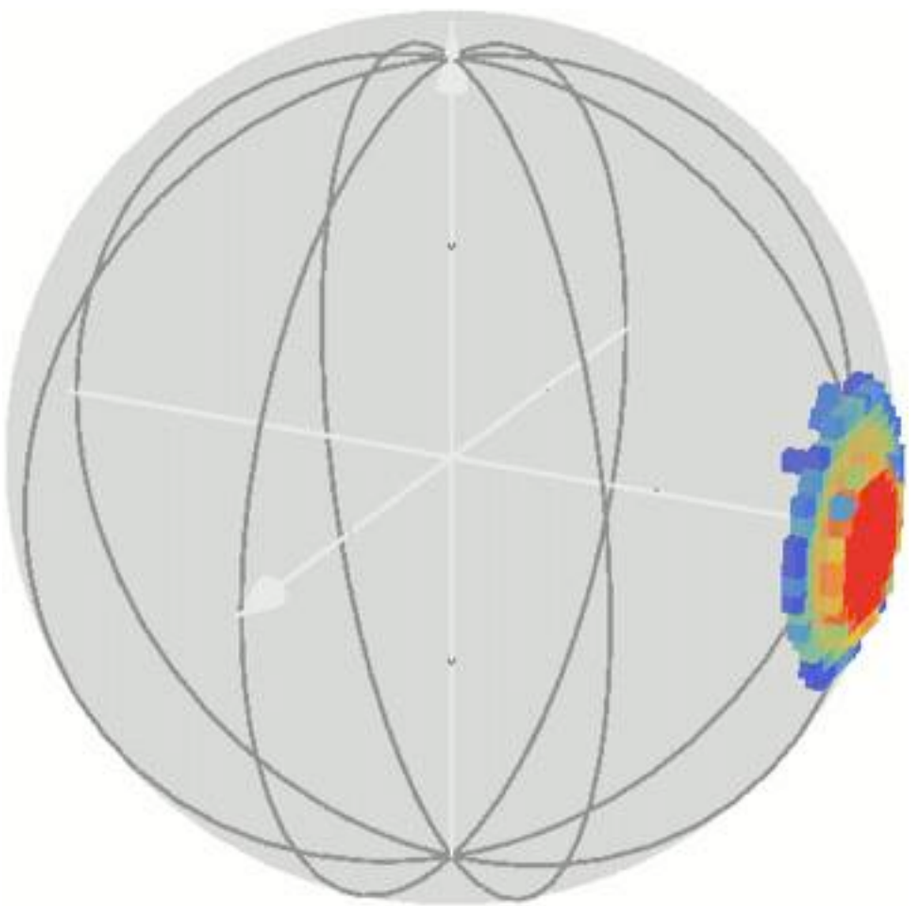
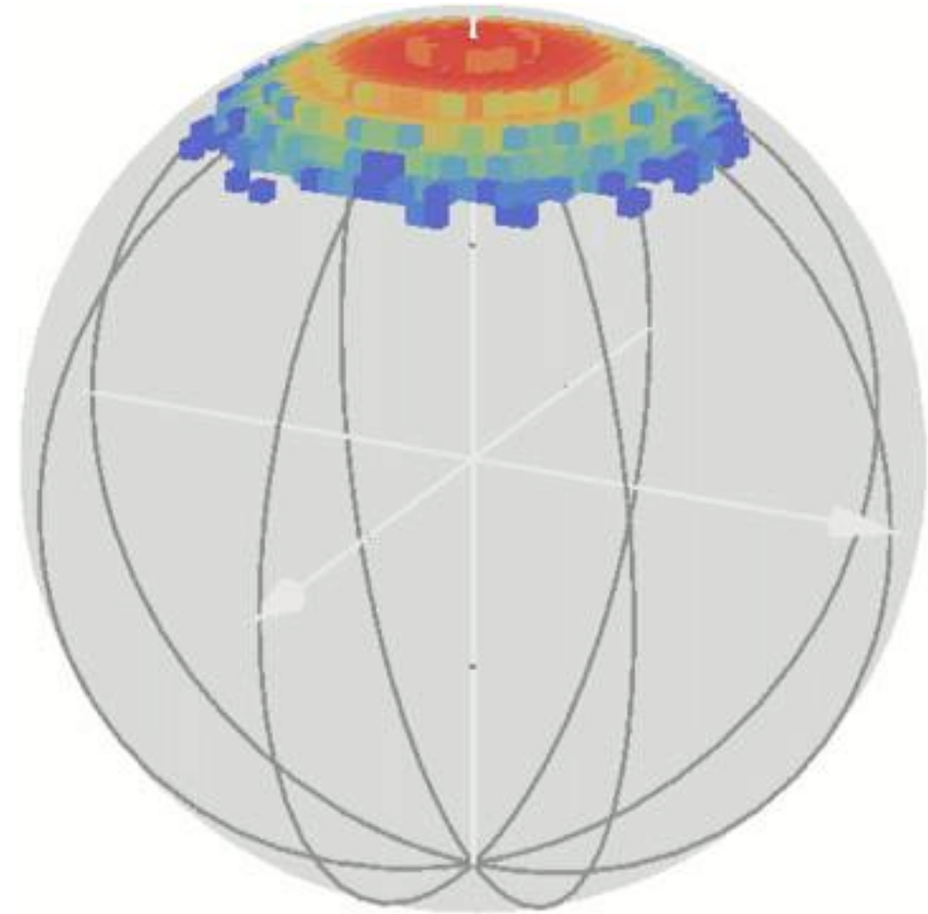
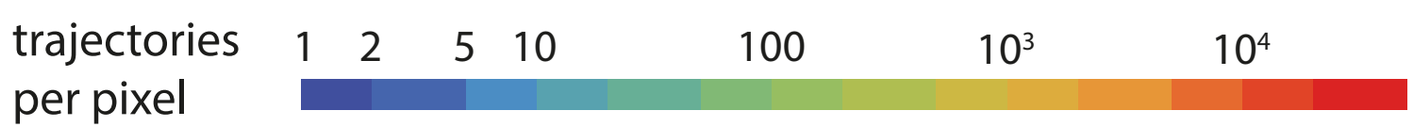




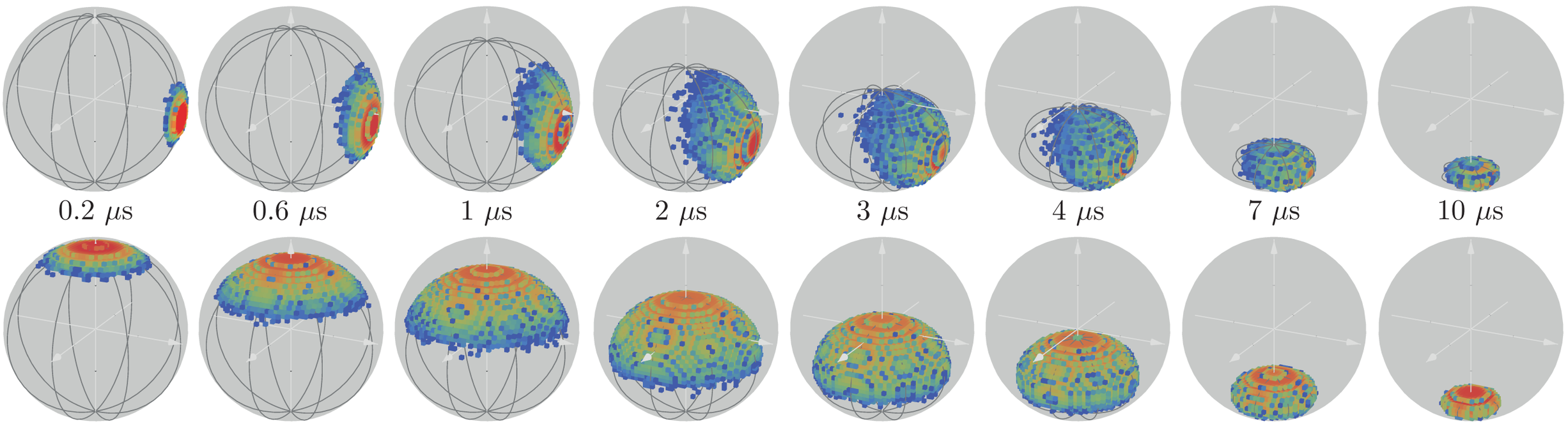
10^6 experiments

$\mathbb{P}(\rho_t)$



[Campagne-Ibarcq *et al.*, PRX 2016]

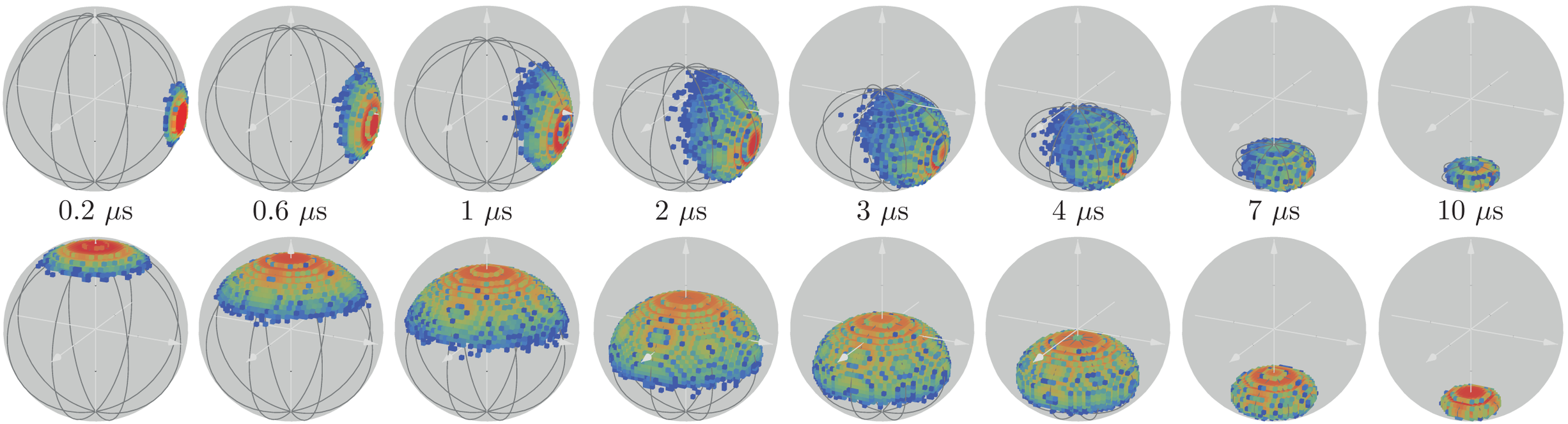
Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

Statistics of trajectories



[Campagne-Ibarcq *et al.*, PRX 2016]

[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

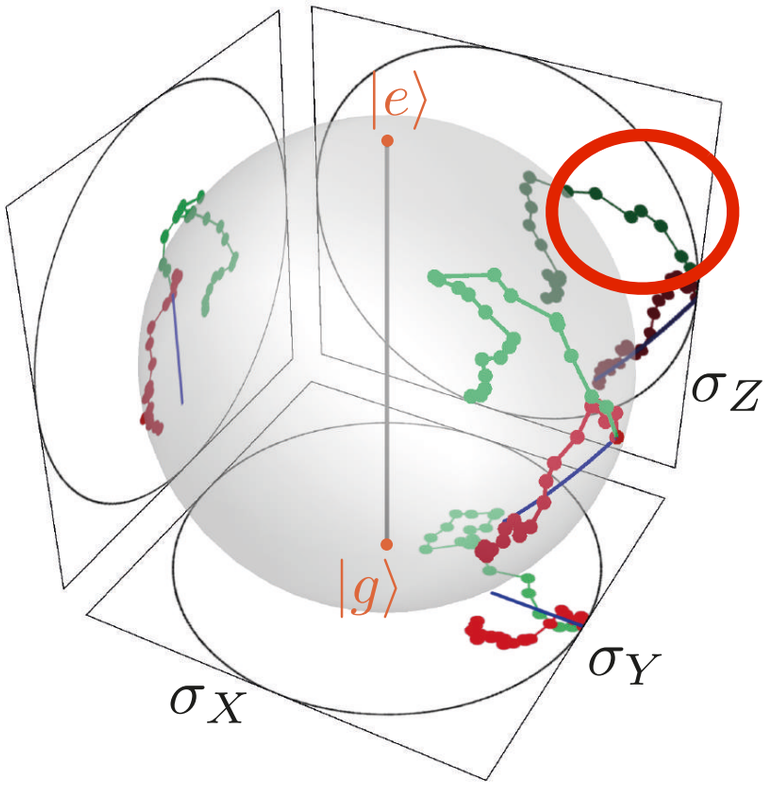
equation of the spheroid

$$\alpha(x^2+y^2)+\alpha^2\left(z+1-\frac{1}{\alpha}\right)^2=1$$

parameter $\alpha(t)=\eta+[\alpha(0)-\eta]e^{\Gamma_1 t}$

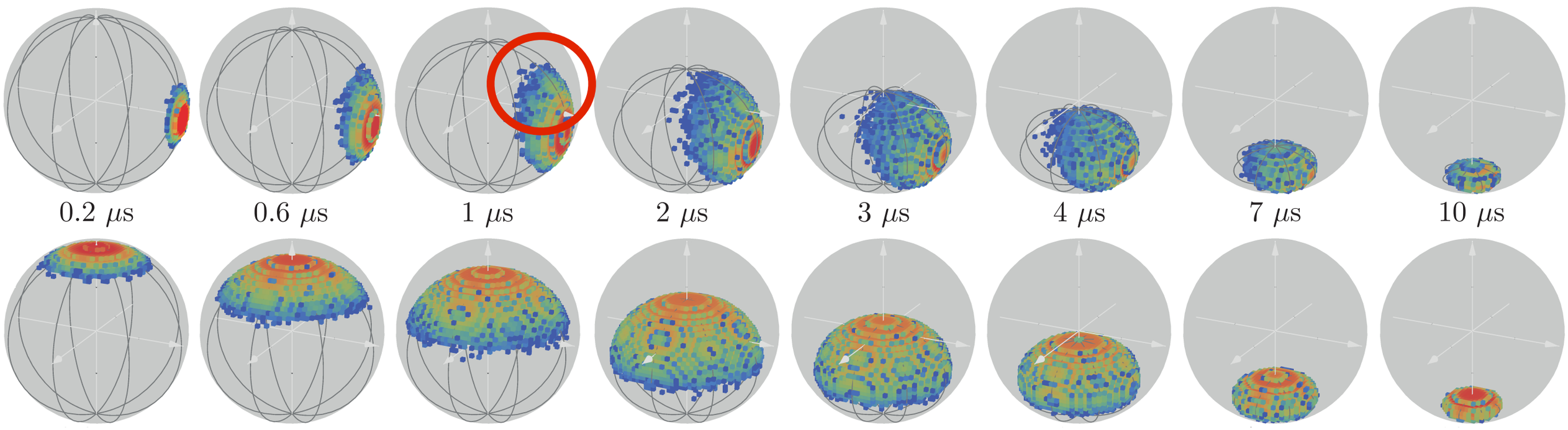
[A.Sarlette and P.Rouchon, Communications in Mathematical Physics 2016]

Counterintuitive trajectories



Energy expectation can **increase** due to the backaction of measuring spontaneously emitted photons

[Bolund and Mölmer, PRA 2014]



[Campagne-Ibarcq *et al.*, PRX 2016]
[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

Parameters Estimation

unknown parameters

$$p = \eta, \rho_0, ...$$

[P.Six et al., arXiv1503.06149v1, 2015]

Parameters Estimation

unknown parameters

$$p = \eta, \rho_0, \ldots$$

hidden Markov problem

$$\rho_k = \frac{\mathbf{K}^p_{y_k}(\rho_{k-1})}{\mathrm{Tr}(\mathbf{K}^p_{y_k}(\rho_{k-1}))}$$

[P.Six et al., arXiv1503.06149v1, 2015]

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guess values

$$p \in \{a, b, ...\}$$

[P.Six et al., arXiv1503.06149v1, 2015]

Parameters Estimation

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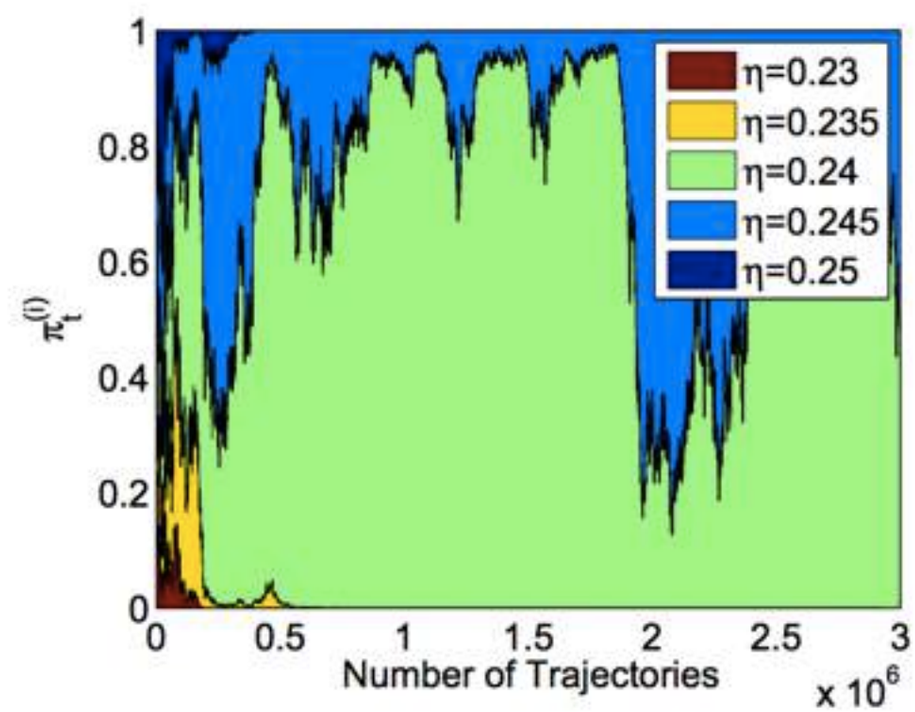
hidden Markov problem

$$\rho_k = \frac{\mathbf{K}^p_{y_k}(\rho_{k-1})}{\mathrm{Tr}(\mathbf{K}^p_{y_k}(\rho_{k-1}))}$$

guess values

$$p \in \{a, b, ...\}$$

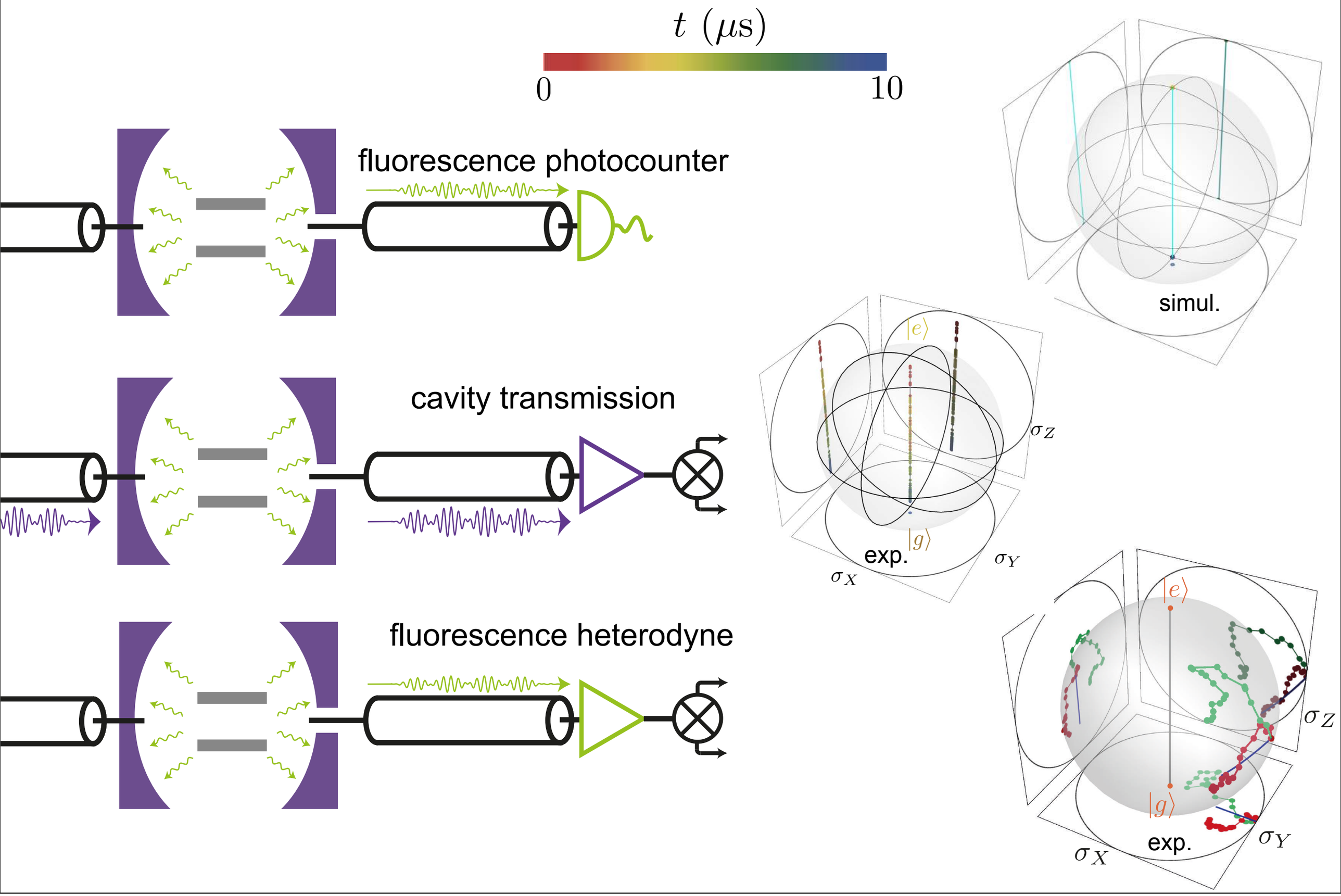
$$\pi^a_k = \frac{\mathrm{Tr}(\mathbf{K}^a(\rho_{k-1}))\pi^a_{k-1}}{\sum_{p'} \mathrm{Tr}(\mathbf{K}^{p'}(\rho_{k-1}))\pi^{p'}_{k-1}}$$



[P.Six et al., arXiv1503.06149v1, 2015]

→ Pierre Six's talk

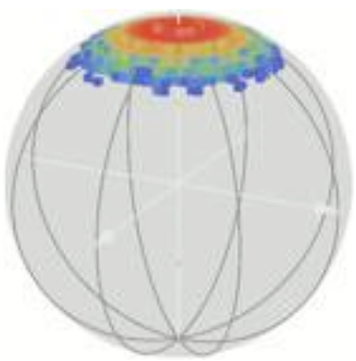
Perspective and conclusion



Interest of Quantum Trajectories

some interesting quantities

$\overline{\rho_t}$ averaged quantum trajectory



$\mathbb{P}(\rho_t)$ probability to get a density matrix at a given time

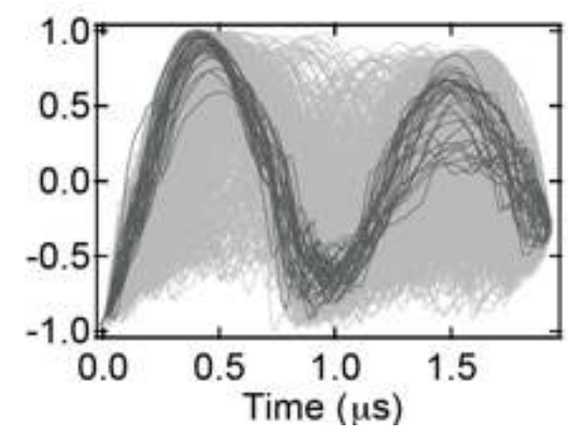
[P. Campagne-Ibarcq, PRX 2016, ENS Paris]

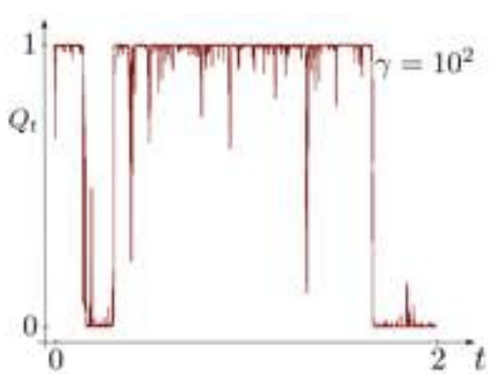
$\mathbb{P}(\{\rho_t\}_{0\leq t\leq T})$ probability distribution to get one given trajectory

$\underset{\{\rho_t\}_t}{\mathrm{argmax}}(\mathbb{P}(\{\rho_t\}_t))$ most likely trajectory

[M. Naghiloo, Nature 2014, St. Louis, USA]

[S.J.Weber, arXiv 2016, Berkley, USA]



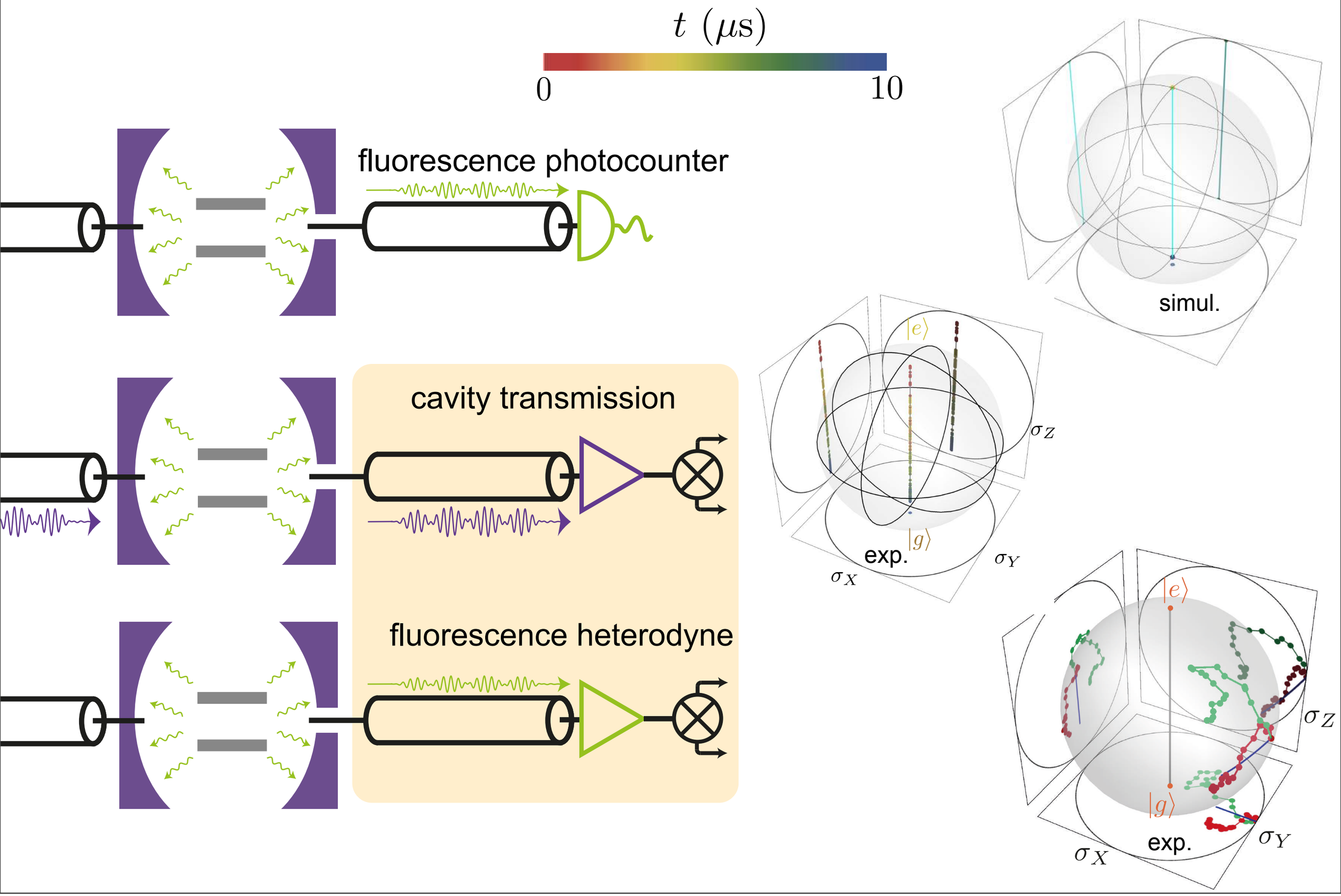


quantum spikes

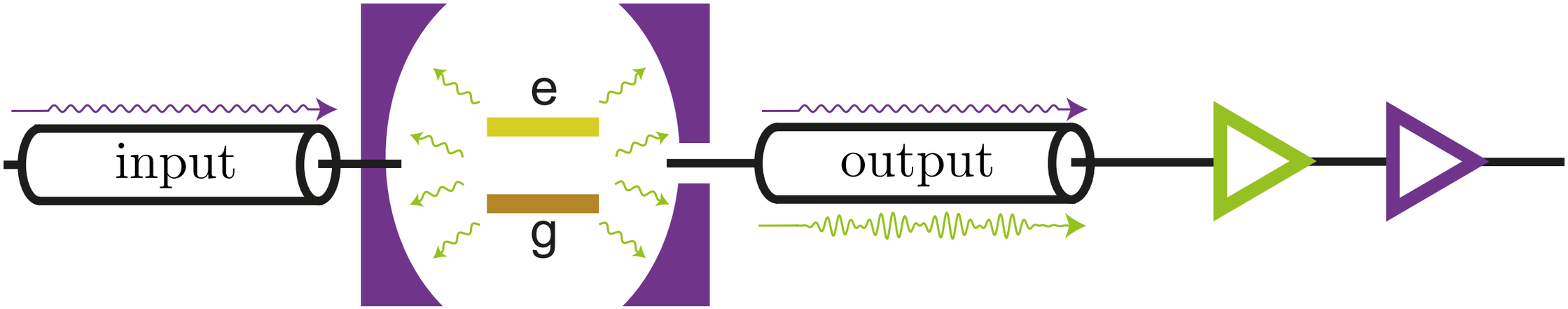
[A.Tilloy, PRA 2015, ENS Paris]

...

Current experiment



Current experiment

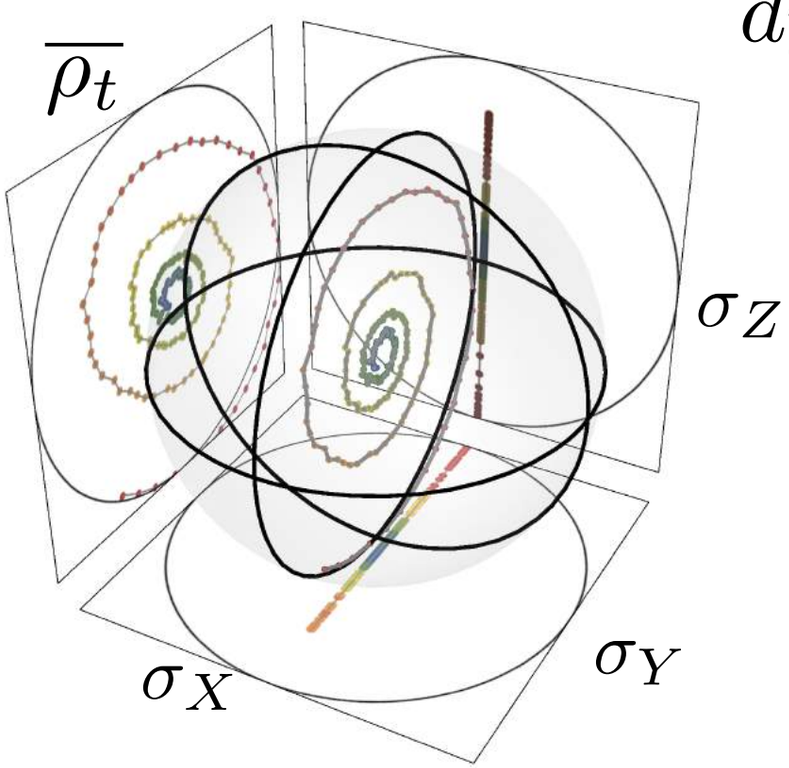


measurement records

$$dy_{t,1} = \sqrt{\eta_{\text{fluo}}\Gamma_1/2}\,\text{Tr}(\sigma_X\rho)dt + dW_{t,1}$$
$$dy_{t,2} = \sqrt{\eta_{\text{fluo}}\Gamma_1/2}\,\text{Tr}(\sigma_Y\rho)dt + dW_{t,2}$$
$$dy_{t,3} = \sqrt{2\eta_{\text{disp}}\Gamma_d}\,\text{Tr}(\sigma_Z\rho)dt + dW_{t,3}$$

average outcome

noise (Wiener)



intermediate regime between Zeno and diffusive

Thanks

Philippe Campagne-Ibarcq (now Yale)

Emmanuel Flurin (now UC Berkeley)

Théau Peronnin

Raphaël Lescanne

Quantic - Theory

Pierre Six

Joachim Cohen

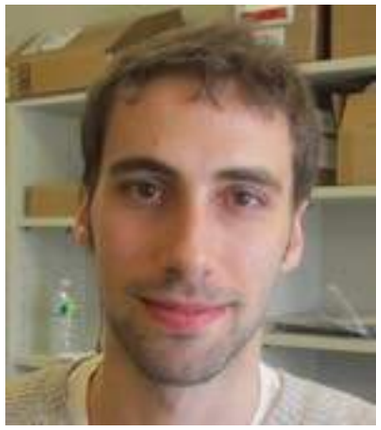
Rémi Azouit

Mazyar Mirrahimi

Pierre Rouchon

Alain Sarlette

Zaki Leghtas



MAIRIE DE PARIS

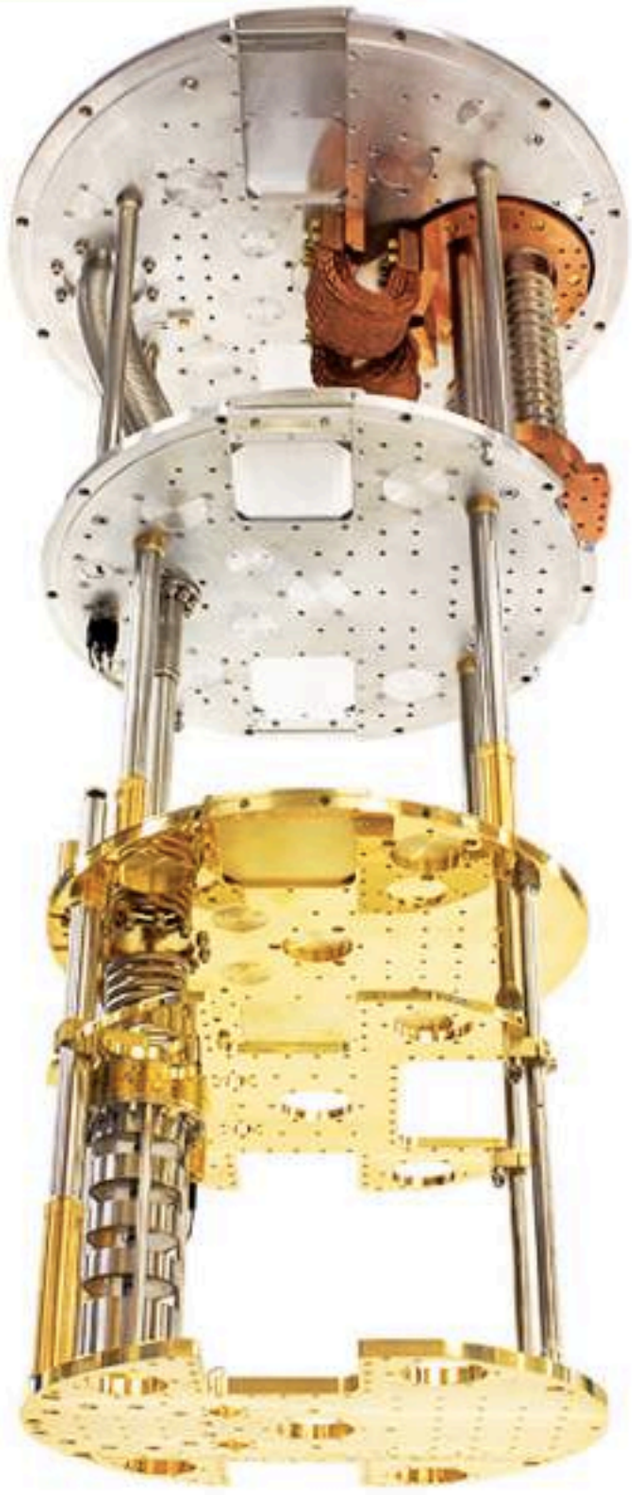
Agence Nationale de la Recherche GIP ANR

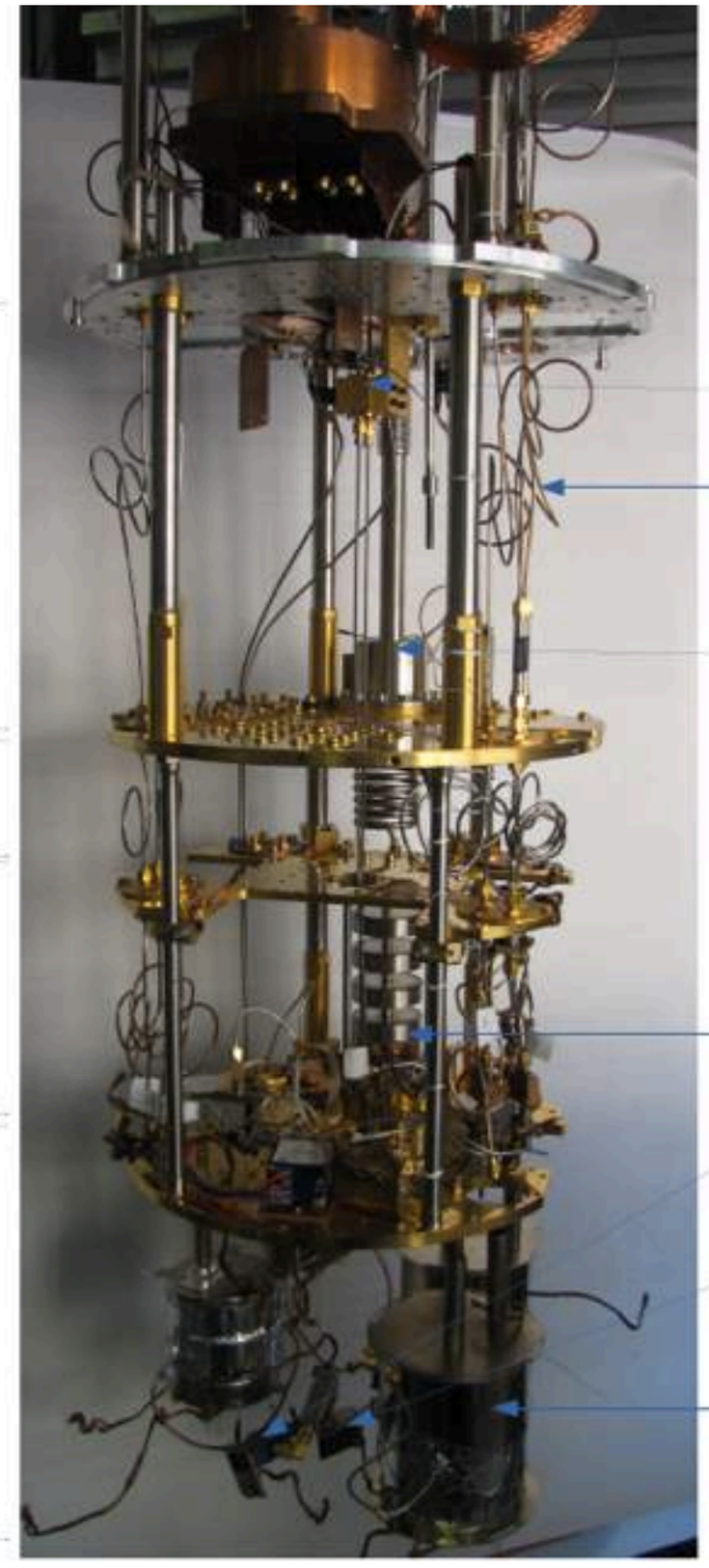
PSL RESEARCH UNIVERSITY

Extra Slides

Dilution Fridge

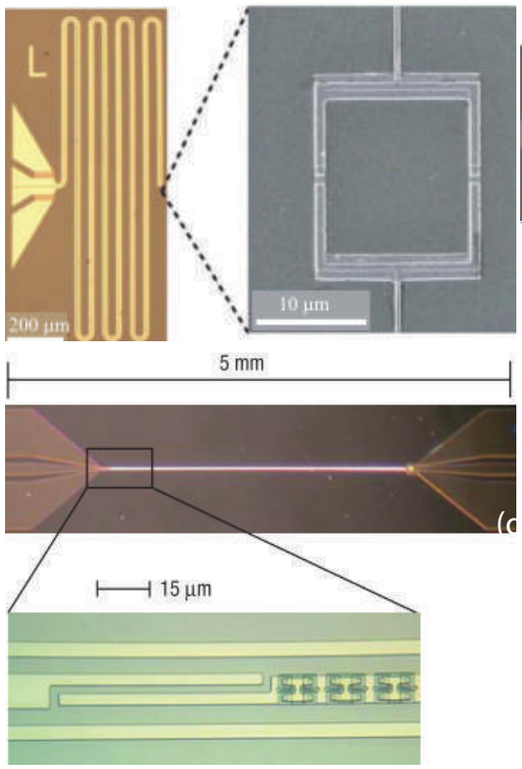
BlueFors CRYOGENICS

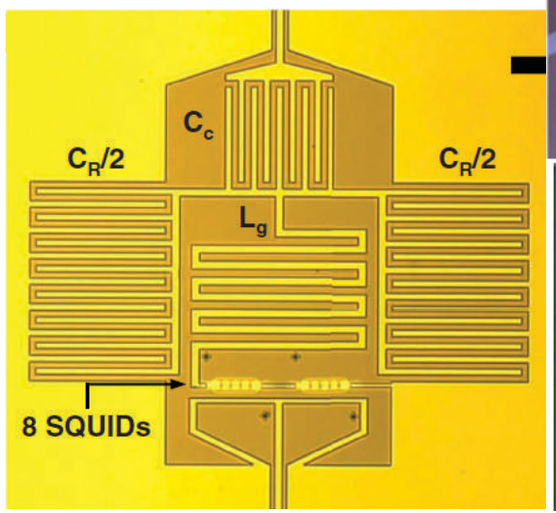


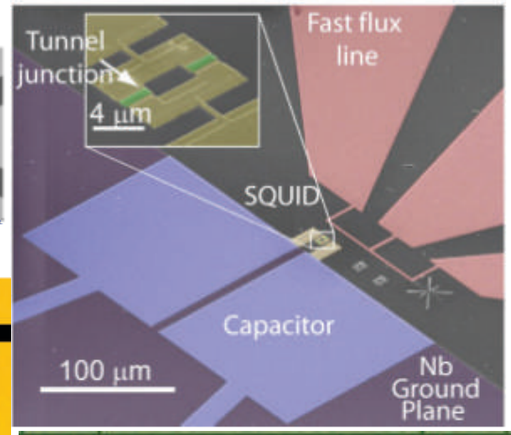


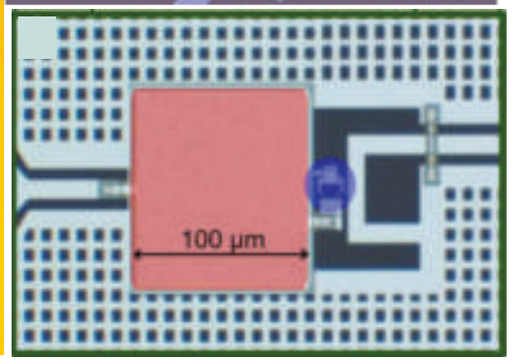
Superconducting amplifiers

Degenerate amplifiers



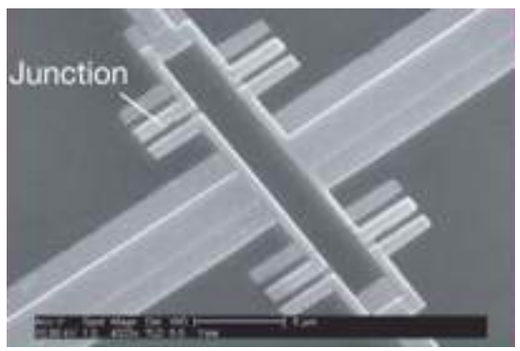


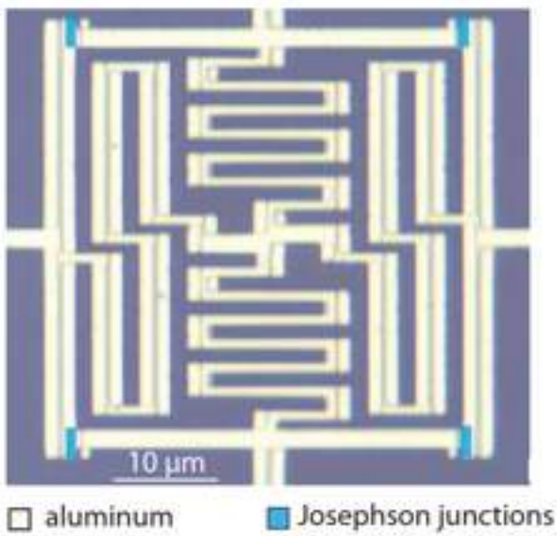




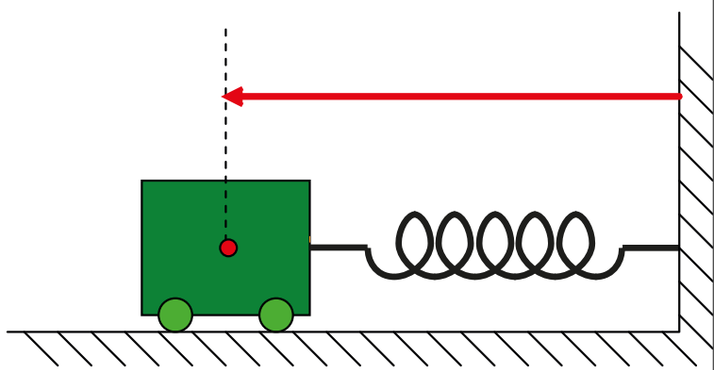
(Bell Labs, 1989)
(NEC Tokyo, 2008)
(Boulder, 2008)
(Yale, 2009)
(Zurich, 2011)
(Berkeley, 2011)
(Santa Barbara,
(Saclay, 2014)

Non degenerate amplifiers

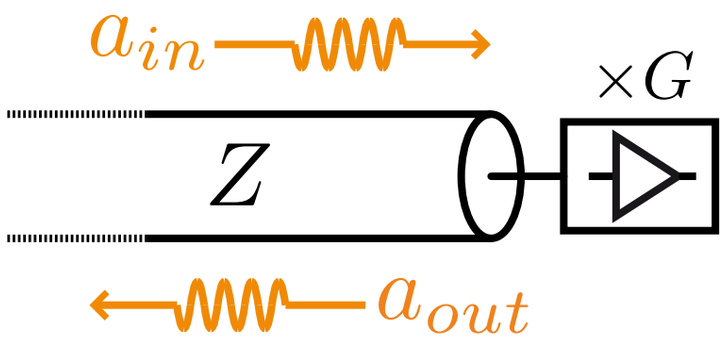




(Yale, 2010)
(ENS Paris, 2012)



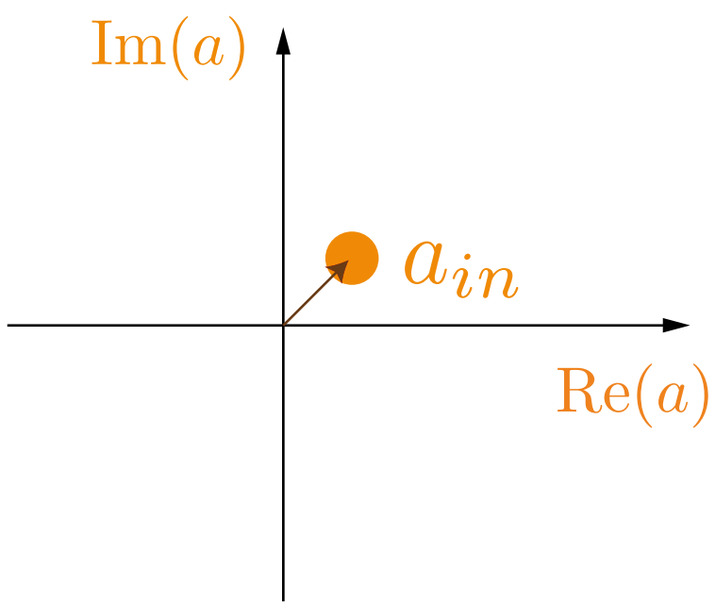
Building your own amplifier

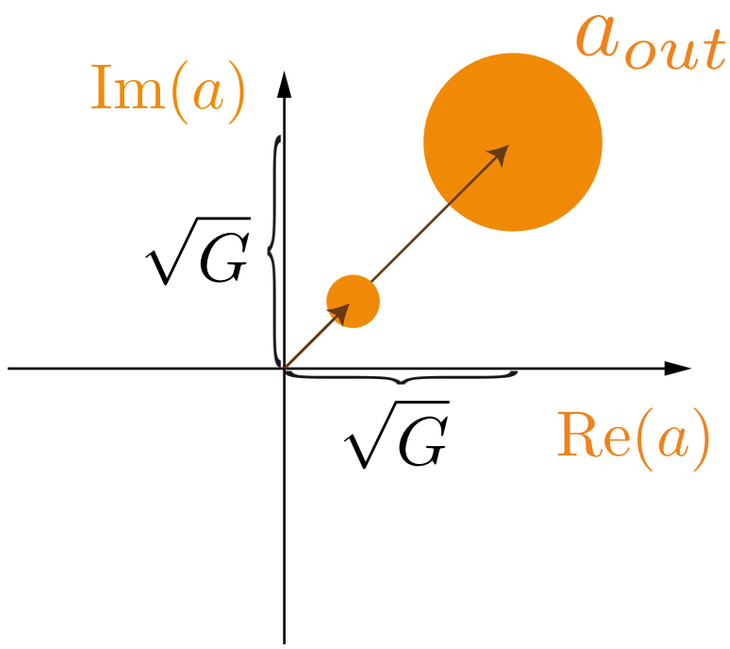


Bosonic mode $[\hat{a}, \hat{a}^\dagger] = 1$

$$\mathrm{Re}(a) = \frac{a + a^\dagger}{2} \qquad \mathrm{Im}(a) = \frac{a - a^\dagger}{2i}$$

Ideally, $\hat{a}_{out} = \sqrt{G}\hat{a}_{in}$





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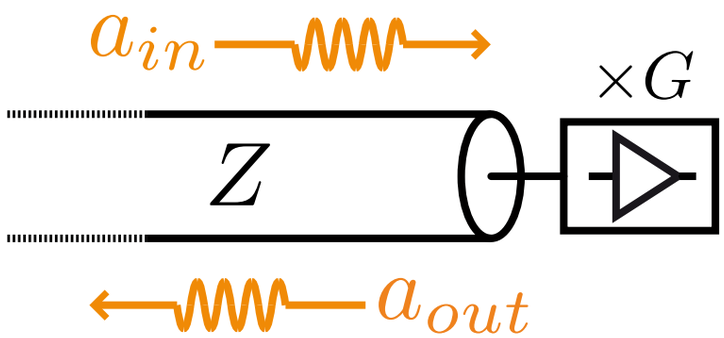

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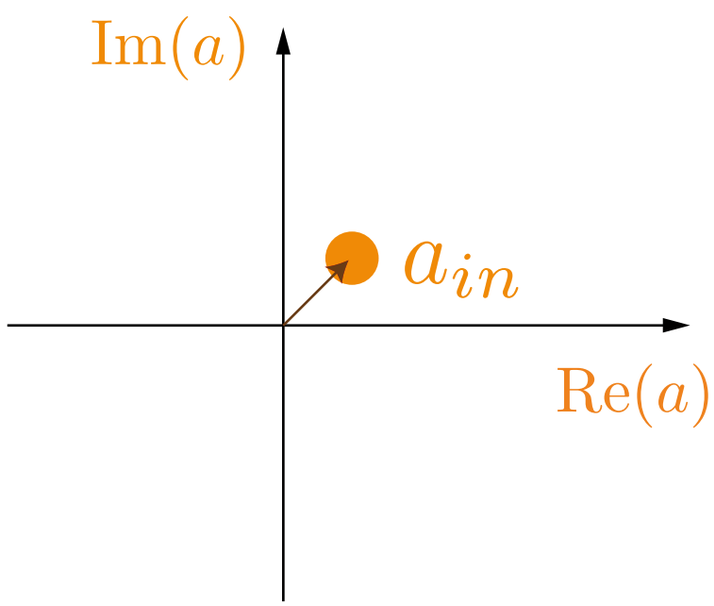
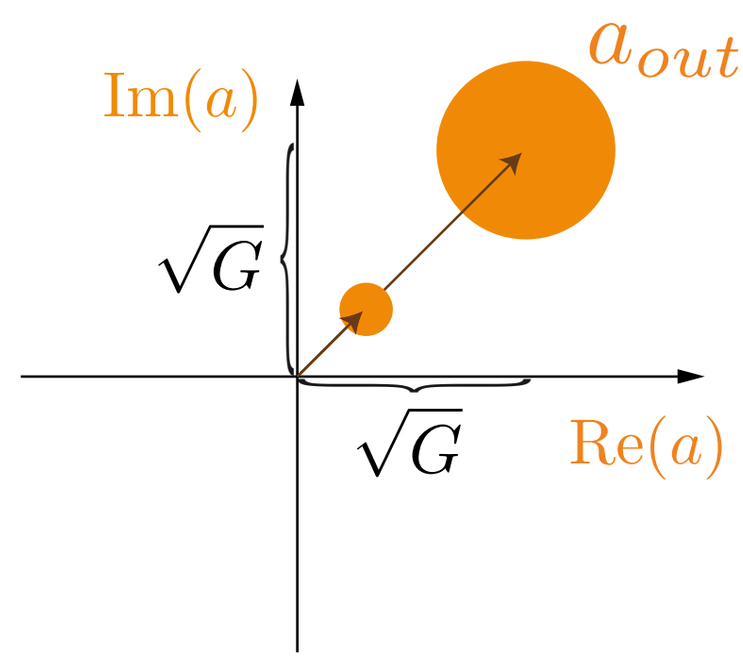
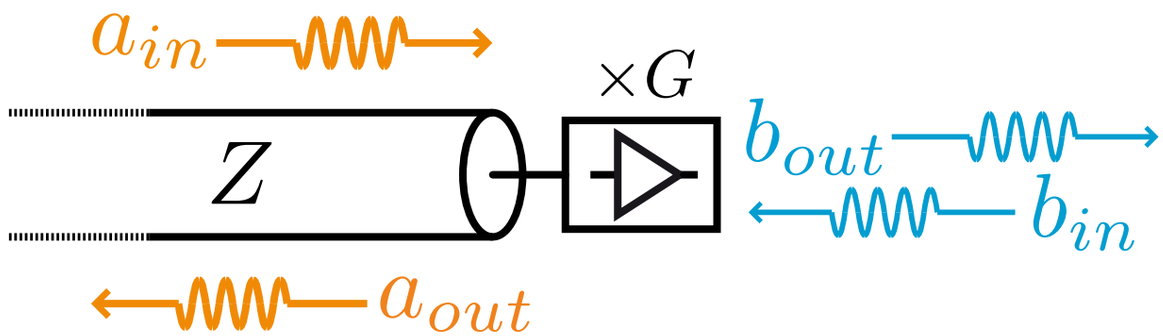




Phase sensitive and phase preserving amplification

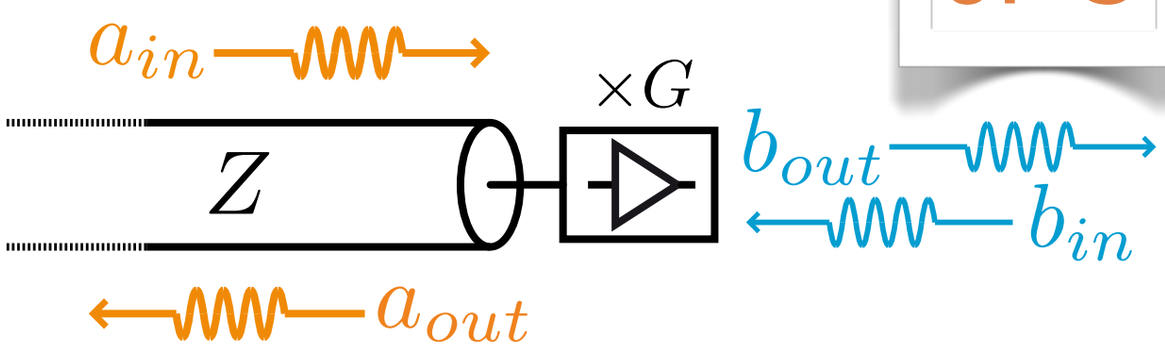
Solution: add an extra degree of freedom to the system



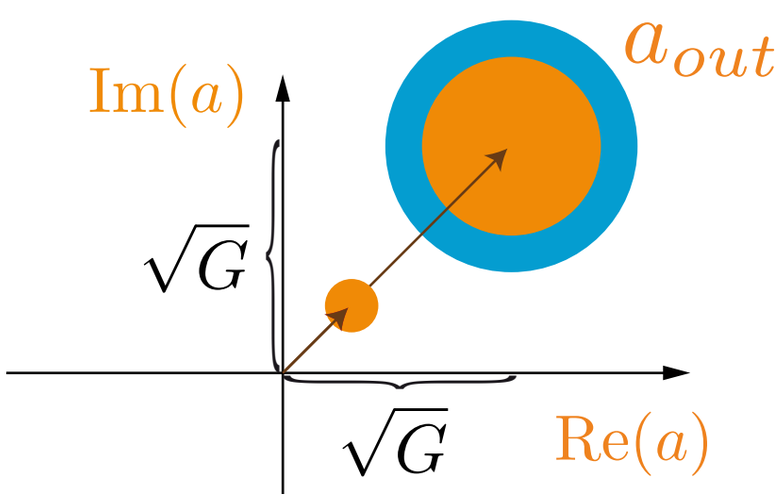
Phase sensitive vs phase preserving amplification

Non-degenerate $\hat{a} \neq \hat{b}$

JPC



$$\hat{a}_{out} = \sqrt{G}\hat{a}_{in} + \sqrt{G-1}\hat{b}^{\dagger}_{in}$$



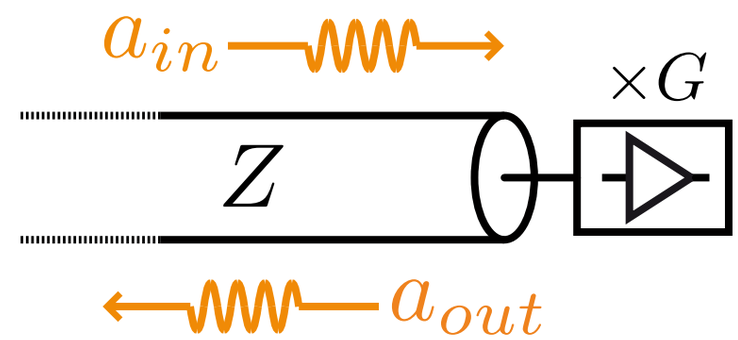
$$\Delta a^2_{out} \geq G\Delta a^2_{in} + (G-1)\frac{1}{2}$$

[Caves, PRD (1982)]

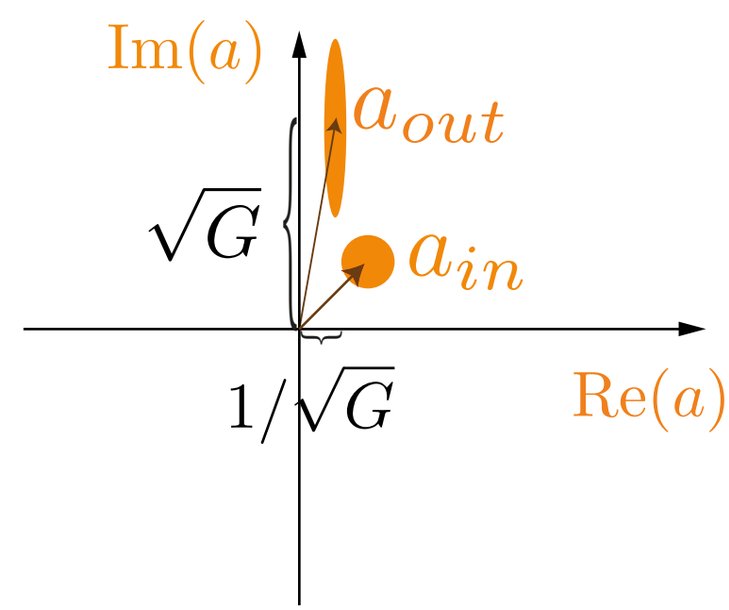
Caves theorem

JPA

Degenerate $\hat{a} = \hat{b}$



$$\hat{a}_{out} = \sqrt{G}\mathrm{Re}(\hat{a}_{in}) + \frac{i}{\sqrt{G}}\mathrm{Im}(\hat{a}_{in})$$



Arbitrarily low noise added on $\mathrm{Im}(a)$

Kraus formalism for parameter estimation

$$\rho_{t+dt} = \frac{\mathbf{K}_{dy_t,dt}(\rho_t)}{\mathrm{Tr}(\mathbf{K}_{dy_t,dt}(\rho_t))}$$

$$\mathbf{K}_{dy,dt}(\rho) = M_{dy,dt}\rho M^\dagger_{dy,dt} + \sum_{\nu=1}^{m}(1-\eta_\nu)dt L_\nu \rho L^\dagger_\nu$$

with $M_{dy,dt} = \mathbf{1} - \left(iH + \sum_{\nu=1}^{m} L^\dagger_\nu L_\nu/2\right)dt + \sum_{\nu=1}^{m}\sqrt{\eta_\nu}dy^\nu L_\nu$