

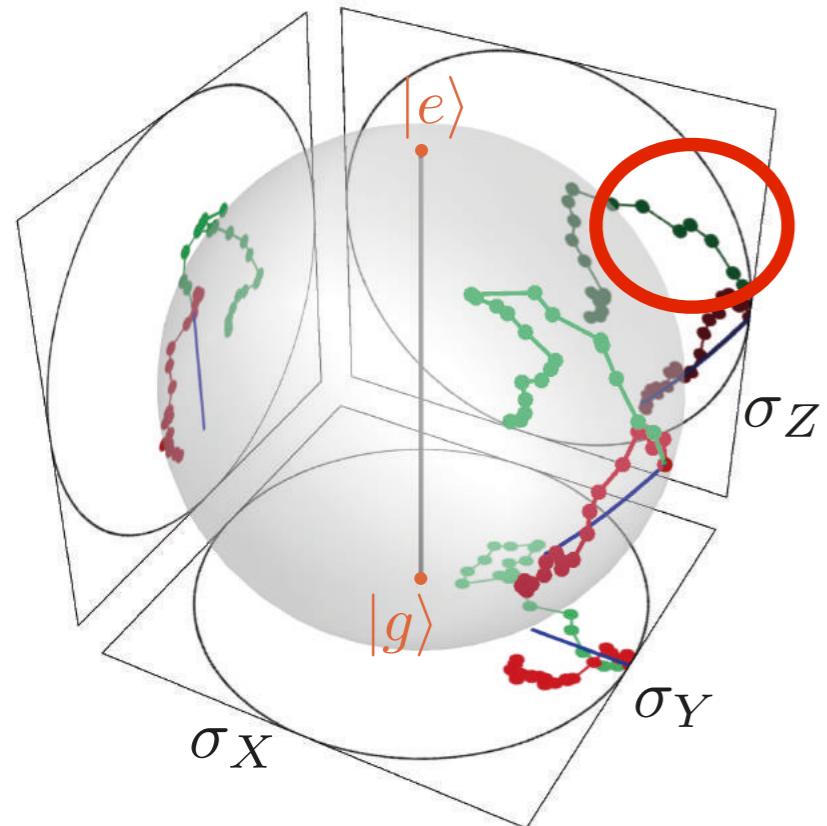


Workshop : Thermodynamics at the single quantum trajectory level ?

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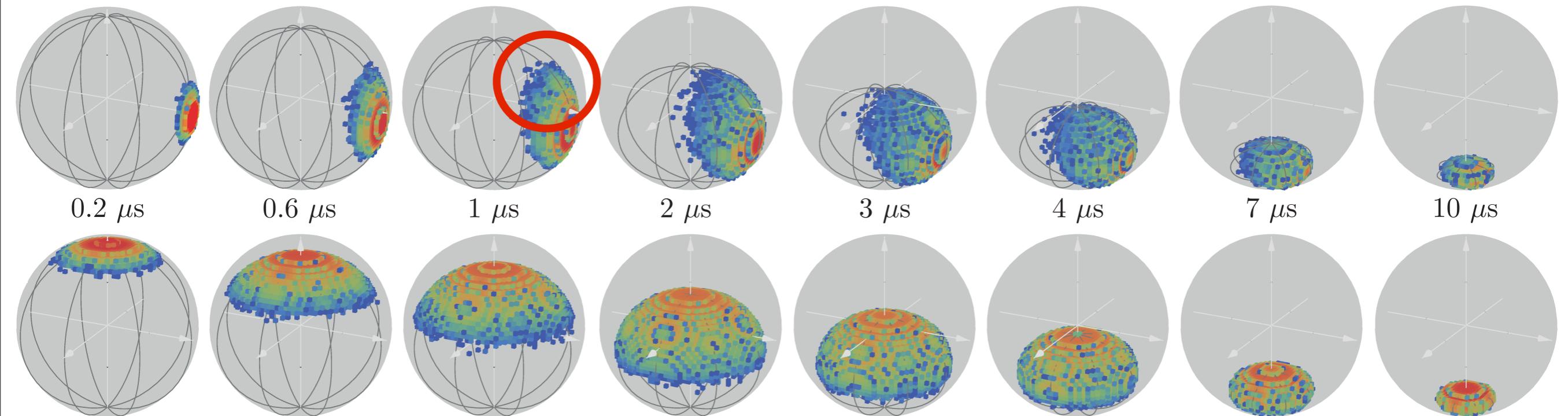
Quantum trajectories, parameter and state estimation
Toulouse 2017

Motivation : Counterintuitive trajectories



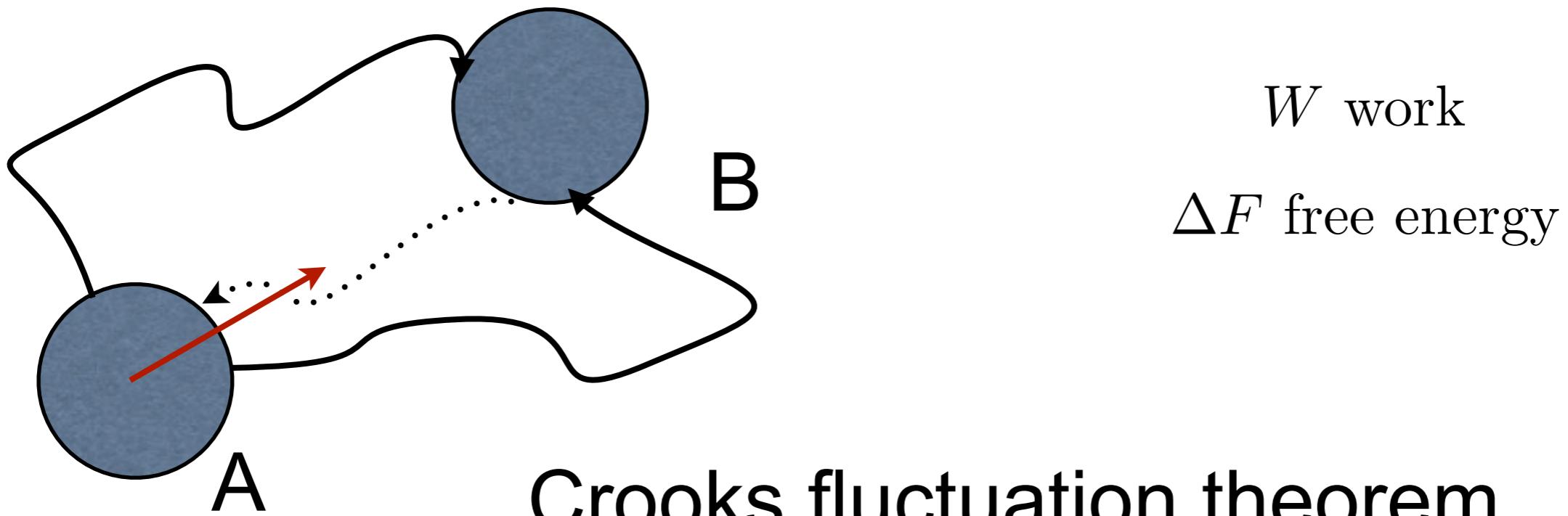
Energy expectation can **increase** due to the backaction of measuring spontaneously emitted photons

[Bolund and M\"olmer, PRA 2014]



[Campagne-Ibarcq *et al.*, PRX 2016]
[Jordan, Chantasri, Rouchon, BH., arxiv:1511:06677]

Classical Stochastic Thermodynamics



$$\frac{P(A \rightarrow B)}{P(A \leftarrow B)} = e^{(W - \Delta F)/k_B T}$$

Jarzynski equality

$$\langle e^{-W/k_B T} \rangle = e^{-\Delta F/k_B T}$$

Quantum thermodynamics

initial state $\rho_0 = \exp(-\beta H_0)/Z_0$

energy $U = \langle H \rangle = \text{Tr}(\rho_t H_t)$

1st principle $dU = \text{Tr}(\rho_t dH_t) + \text{Tr}(d\rho_t H_t) = \delta W + \delta Q$

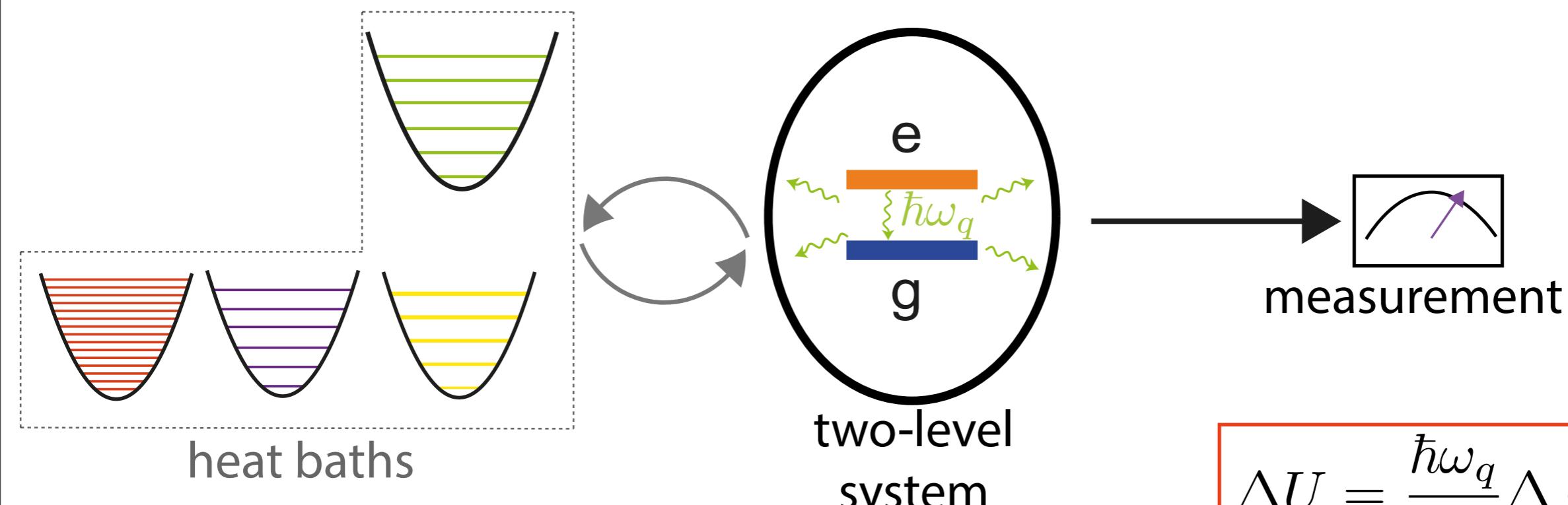
Two projective measurements

$$\left\langle e^{-W/k_B T} \right\rangle = e^{-\Delta F/k_B T}$$



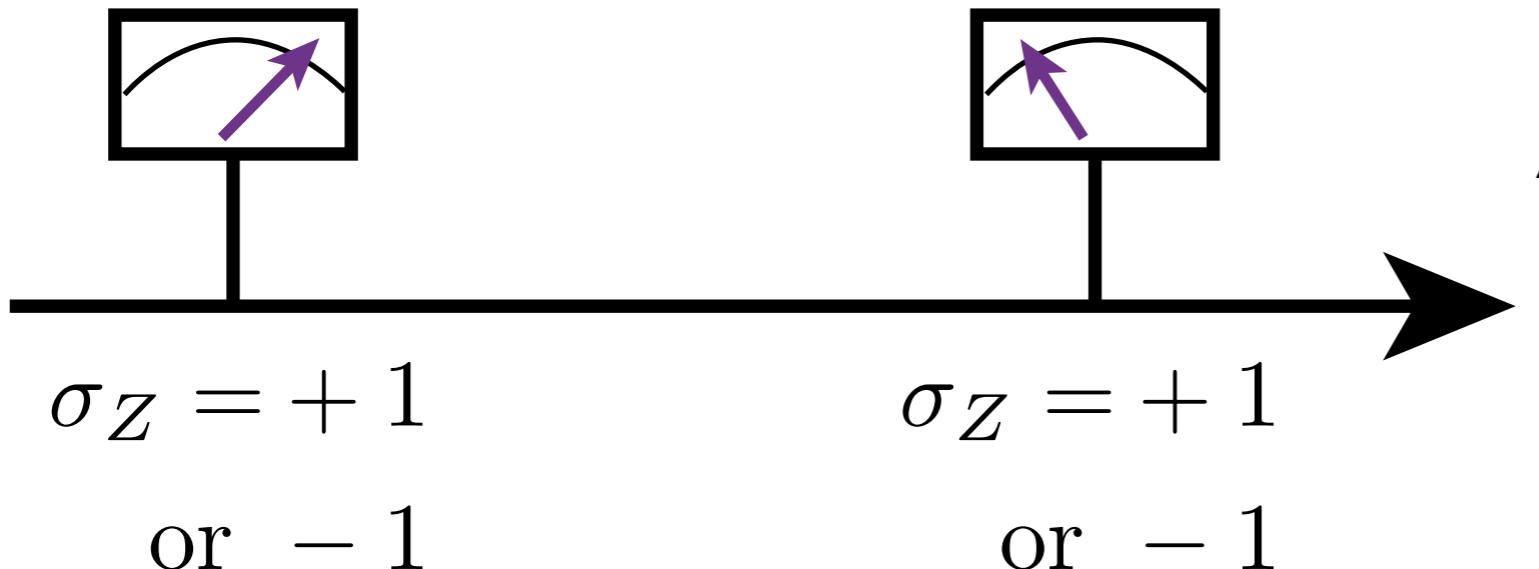
A Quantum Fluctuation Theorem by Jorge Kurchan 2001

Quantum thermodynamics



$$\Delta U = \frac{\hbar\omega_q}{2} \Delta \langle \sigma_Z \rangle$$

Two projective measurements



$$\Delta U = -\hbar\omega_q, 0 \text{ or } \hbar\omega_q$$

A Quantum Fluctuation Theorem by Jorge Kurchan 2001
[Shuoming An et al., Nature Physics 11, 193–199, 2015]

Zurek !

Quantum work and the thermodynamic cost of quantum measurements
Sebastian Deffner Juan Pablo Paz and Wojciech H. Zurek, USA, 2016

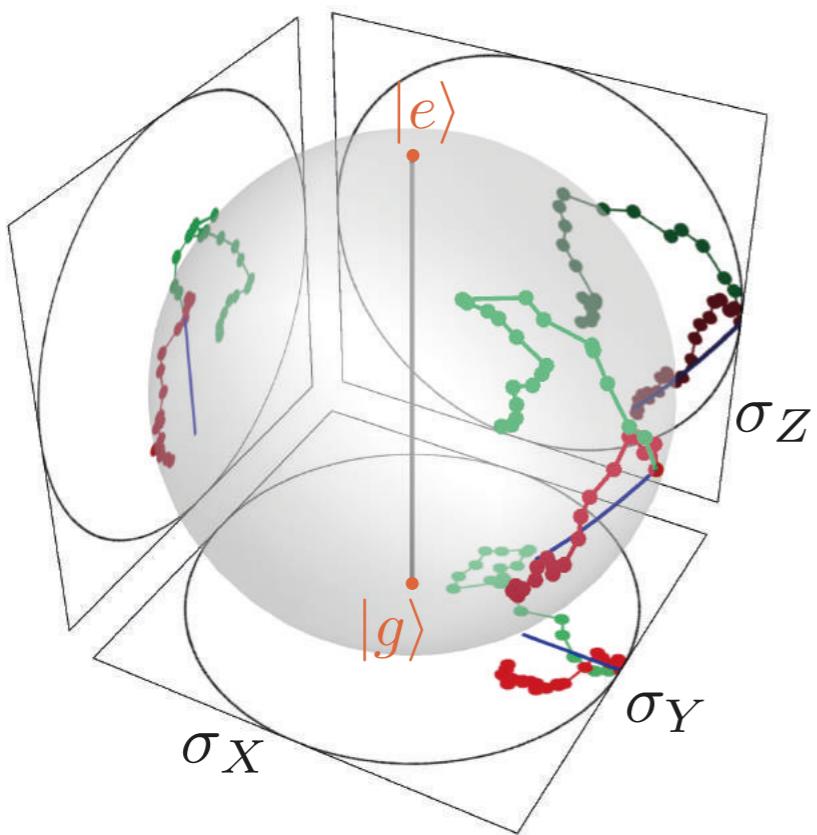


During each measurement you have to take into account the **work due to the back-action of measurement**

$$W = \langle n_0 | U_\tau^\dagger H_\tau U_\tau | n_0 \rangle - \epsilon(n_0, \lambda_0)$$

A Quantum Fluctuation Theorem by Jorge Kurchan 2001
[Shuoming An et al., Nature Physics 11, 193–199, 2015]

Quantum thermodynamics

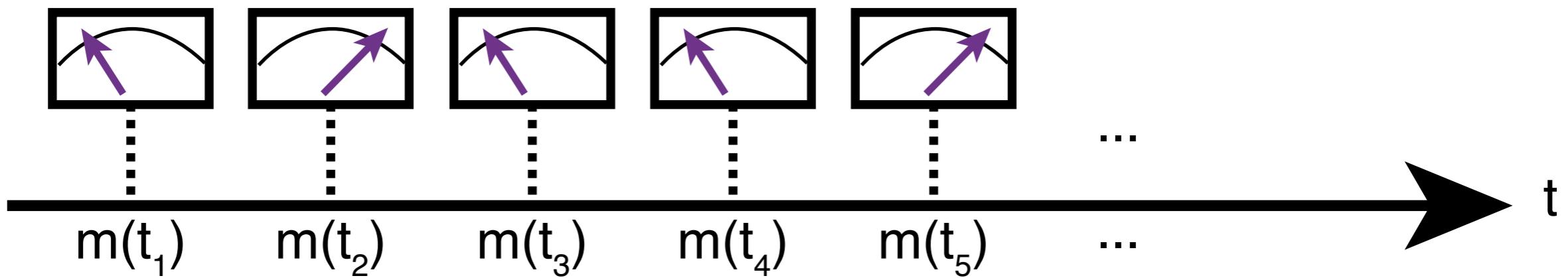


Is there a Jarzynski or Crook relation ?

What is work ?

What is heat ?

Continuous weak measurement



A Quantum Fluctuation Theorem by Jorge Kurchan 2001

Open question

Thermodynamics of weakly measured quantum systems

Jose Joaquin Alonso, Eric Lutz, and Alessandro Romito, 2016

$$d\rho_t = -\frac{i}{\hbar}[H, \rho_t]dt + \sum_{i=1}^m \mathcal{D}_i(\rho_t)dt + \sum_{i=1}^m \sqrt{\eta_i} \mathcal{M}_i(\rho_t)dW_{t,i}$$

$$d\rho_t = \delta W[\rho_t]dt + \delta Q[\rho_t]dt \quad + \text{Jarzynski equality}$$

unitary non unitary

Quantum work and the thermodynamic cost of quantum measurements

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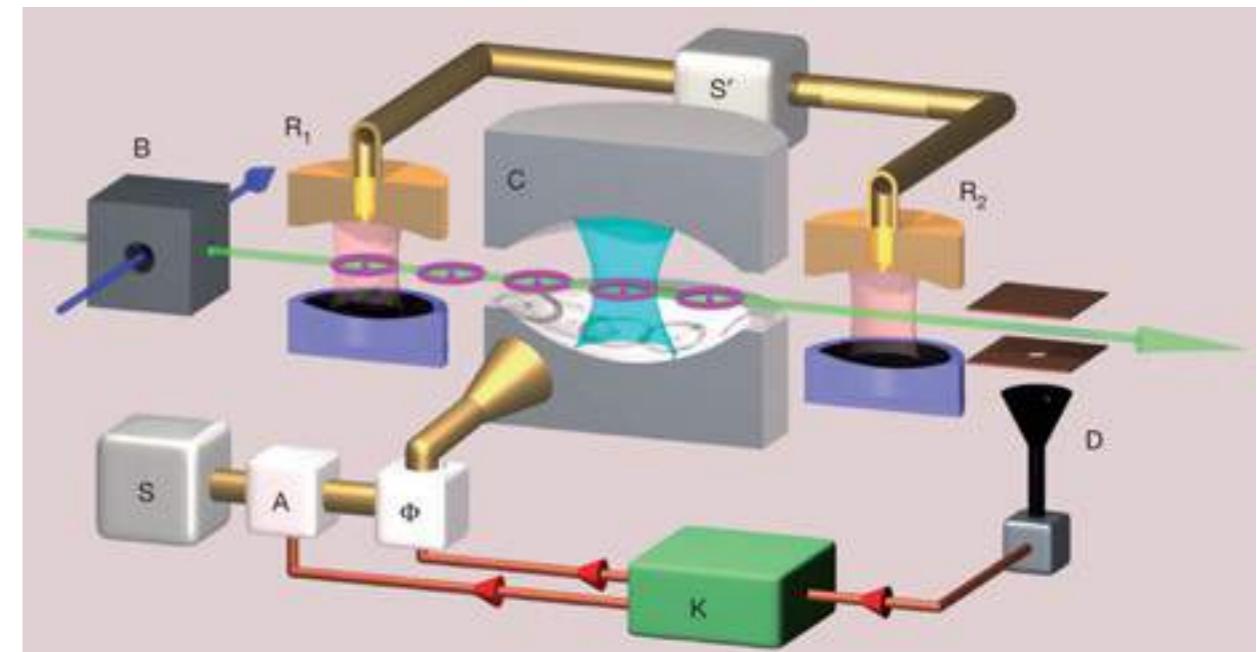
Open question

Quantum-trajectory approach to the stochastic thermodynamics of a forced harmonic oscillator

Jordan M. Horowitz, 2012

Stochastic Schrödinger Equation Formalism

$$d|\psi_t\rangle = dt \left(-i H(f_t) - \frac{gr_0}{2} \bar{a}_t^\dagger \bar{a}_t - \frac{gr_1}{2} \bar{a}_t \bar{a}_t^\dagger + \frac{gr_0}{2} \langle \bar{a}_t^\dagger \bar{a}_t \rangle_t + \frac{gr_1}{2} \langle \bar{a}_t \bar{a}_t^\dagger \rangle_t \right) |\psi_t\rangle + dN_t^- \left(\frac{\bar{a}_t^\dagger |\psi_t\rangle}{\sqrt{\langle \bar{a}_t \bar{a}_t^\dagger \rangle_f}} - |\psi_t\rangle \right) + dN_t^+ \left(\frac{\bar{a}_t |\psi_t\rangle}{\sqrt{\langle \bar{a}_t^\dagger \bar{a}_t \rangle_t}} - |\psi_t\rangle \right).$$



$$dQ_t[\psi] = \omega(dN_t^- - dN_t^+).$$

$$U_t[\psi] = \langle \psi_t | H(f_t) | \psi_t \rangle.$$

$$dW_t[\psi] = dU_t[\psi] - dQ_t[\psi],$$

+ Jarzynski equality

Perfectly measure the environment

Open question

Stochastic thermodynamics in the quantum regime

Cyril Elouard, Alexia Affèves, and Maxime Clusel, 2015

Again stochastic Schrödinger equation formalism

$$d|\psi(t, \vec{\mu})\rangle = -\frac{i dt}{\hbar} \hat{H}_{\text{eff}}(t) |\psi(t, \vec{\mu})\rangle + d\hat{D}(t) |\psi(t, \vec{\mu})\rangle$$

Trace out the environment



$$dU_t[\psi] = \delta W[\psi] + \delta Q_c[\psi] + \delta Q_q[\psi]$$

quantum heat

work?

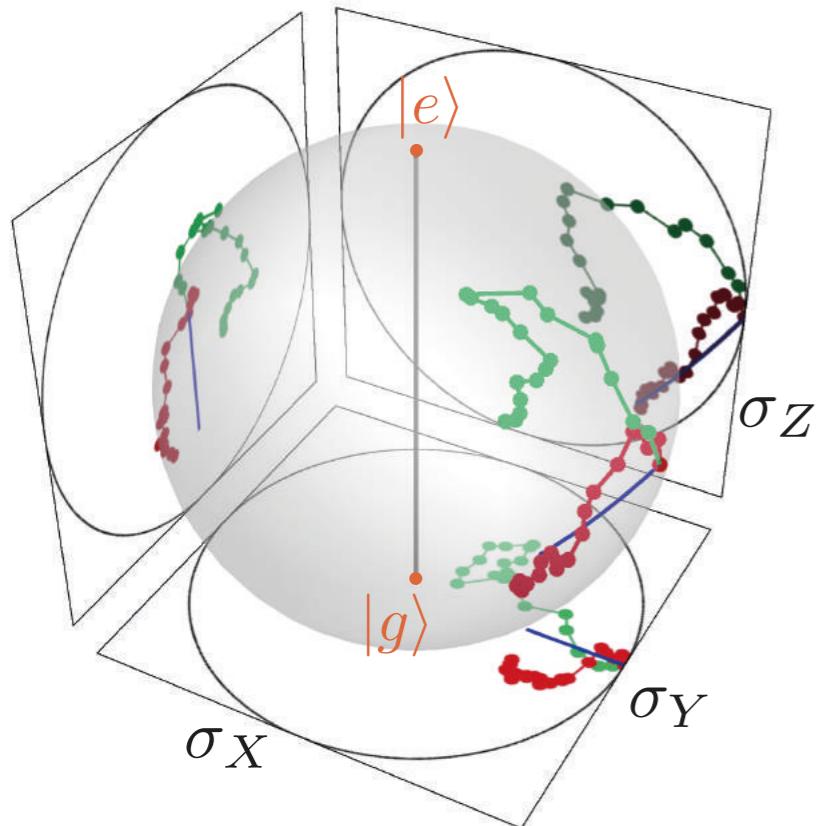
heat?

interaction with the detector

mutual information

energy relocalisation?

Open question



How to quantify the occurrence of unlikely single quantum trajectories in this context ?

Is thermodynamics relevant ?

context :

- Open system
- Weak continuous measurement
- We cannot monitor perfectly the environment
- We don't have access to the reverse trajectory

References

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